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AEROELASTIC AND STRUCTURES RESEARCH LABORATORY
MASSACHUSETTS INSTITUTE OF TECHNOLOGY

TECHNICAL REPORT 73-6

THE INFLUENCE OF AERODYNAMIC HEATING ON THE STRUCTURAL DESIGN OF HIGH-SPEED AIRCRAFT

PART IX

DEVELOPMENT OF STRUCTURAL DESIGN CRITERIA FOR AIRCRAFT SUBJECTED TO AERODYNAMIC HEATING

BY

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FOR THE

BUREAU OF NAVAL WEAPONS

DEPARTMENT OF THE NAVY

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ABSTRACT

The influence of aerodynamic heating on the structural strength, stiffness and life design of high-speed aircraft is discussed. Critical design conditions are suggested for the strength and stiffness design of airframes experiencing elevated temperatures. Fatigue and creep design specifications are discussed. Safety factor requirements are recommended.

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NOMENCLATURE

a_x	probability distribution of aircraft strength
b	span
c	chord or length; velocity of sound, ft/hr
c_p	specific heat
f_x	probability per flight hour of exceeding a load x
$\hat{f}(\sigma_u)$	distribution of root mean square gust velocity
h	altitude; heat transfer coefficient, BTU/ft^2 ^{o}R hr
m	number of missions
n	normal load factor
n_k	given value of normal load factor
q	heat flux, BTU/hr ft^2
r	recovery factor
t	time
u	airstream velocity, ft/hr
v	velocity
x	cartesian coordinate; distance from transition point, ft.
y	a random variable
K	thermal conductivity, BTU/ft ^{o}R hr.
M	Mach number
M_i	i^{th} mission

P	probability
$P[(\ )/(\ )]$	conditional probability
$P[(\ ),(\ )]$	joint probability
P_r	Prandtl number
Q	heat impulse
T	temperature, ^{o}R
T_{aw}	adiabatic wall temperature ^{o}R
T_{M_i}	flight time of mission M_i
V_{cruise}	cruise velocity
V_{dive}	dive velocity
V_e	equivalent airspeed
V_L	limit airspeed
a	thermal diffusivity
γ	specific heat ratio
δ	variation about mean value; thickness
ε	total hemispherical emissivity
μ	absolute viscosity, lb sec/ft^2
ν	kinematic viscosity, ft^2/hr
ρ	density, slugs/ft^3
σ	standard deviation; Stefan-Boltzmann constant
$\phi(\omega)$	power spectrum
ω	frequency
∇^2	Laplacian operator

Subscripts

aw	adiabatic wall
i	associated with i^{th} mission; i^{th} term in series
o	mean value; sea level
s	stratosphere
t	troposphere
w	of the wall
*	tropopause

CHAPTER I

OBJECTIVE OF STUDY

The high operational speeds attained by modern flight vehicles introduces the problem of aerodynamic heating. As a result, future designs must account for such undesirable phenomena as thermal stresses, reduction of material properties with elevated temperature, and creep. Through these phenomena two new parameters, time and temperature, enter the design picture. The question immediately arises as to what criteria would be appropriate upon which to base the structural design of such aircraft. The current airplane strength and rigidity specifications are not entirely adequate in this regard; thus there is a need for new criteria that will provide the basis for the structural design of aircraft subjected to the effects of aerodynamic heating. The objective of this report is to provide information that would be useful in the establishment of new structural design criteria.

CHAPTER II

INTRODUCTION

It is obvious that the formulation of a basic structural design criteria encompassing all flight vehicles subjected to aerodynamic heating is a formidable task. The introduction of the two new parameters of time and temperature makes it difficult to predict generalized critical flight loading conditions such as is done in the familiar V - n diagram, as the effects of time and temperature (e.g., thermal stresses) depend upon the particular structural configuration and flight history. Aerodynamic heating introduces several new modes of structural failure such as creep; the behavior of these modes of failure under combined thermal and external loading must be determined to facilitate the promulgation of appropriate design criteria. In addition, in contrast to present design criteria, which are based upon years of accumulated operational experience, there is to date very little operational experience to help formulate and substantiate criteria for vehicles whose structures will be subjected to the effects of aerodynamic heating.

To graphically illustrate the structural design complexities introduced by aerodynamic heating, consider the design of a flight vehicle operating in the flight regime where aerodynamic heating of significant magnitude is encountered. The individual who will have a need for and utilize the structural design criteria is the structural design engineer. Figure 2.1 illustrates the problems that the structural designer is confronted with in the design of a high speed aircraft. The diagram shows how the addition of temperature affects each phase of structural design. The standard design analysis methods utilized in the design of airplanes not subjected to aerodynamic heating are represented by the blocks to the right in

Figure 2. 1. When the lines are connected from the left side of Figure 2. 1, this represents the thermal design analysis. This illustrates how the standard methods must be modified to include these thermal effects. Some particular problems that are affected by the inclusion of temperature are stress analysis, divergence and dynamic loading. Figure 2. 1 indicates the areas of structural design that any appropriate criteria must accommodate, and thus helps to define the scope of the criteria. For example, the criteria must allow for the direct influence of the temperature time history on new modes of structural failure (such as creep) and the indirect influence of the temperature distribution on strength failure through the mechanism of thermal stresses introduced by the temperature distribution. These areas of structural integrity will be discussed in detail in the next chapter. Thus, before structural design criteria applicable to flight vehicles encountering elevated temperatures can be established, three problem areas must be considered. First, the critical conditions currently selected to demonstrate the structural integrity of the structure with regard to a certain mode of failure must be re-examined with consideration given to the consequences that aerodynamic heating might have on this particular mode of failure. For example, as indicated above, stresses due to the temperature distribution in the structure influence the magnitude of the external loading that can be applied to the structure without resultant failure of the structure due to strength deficiency. Second, where possible, design conditions should be established for new modes of failure that result from elevated temperature. Third, the applicability of a safety factor, either in the form of an all inclusive factor such as is applied at present to the limit load, or in the form of an equivalent safety factor applied to each particular mode of failure must be investigated. This study investigates these three problem areas.

As such a broad range of vehicles must be considered, and as generalized mission studies giving, for example, occurrence rates of various load levels for a given class of flight vehicle are not available, the usefulness of an analytical approach to the problem of establishing a structural

design criteria for heated vehicles is limited. Hence the approach utilized in this study is partially one of synthesis. Various methods of establishing design criteria have been investigated, and those felt to be the most applicable are discussed. Conclusions are drawn and recommendations are made concerning selection of design conditions and application of safety factors.

CHAPTER III

STRUCTURAL INTEGRITY OF HEATED FLIGHT VEHICLES

3.1 Introduction

Before any consideration can be given to the selection of critical structural design conditions or to the applicability of safety factors in the structural design of heated flight vehicles, it is necessary to define the ways in which a flight vehicle may prove structurally inadequate during its lifetime of operation. This determines the basic areas of possible structural deficiency in which the design criteria must provide requirements to insure adequate structural integrity, and makes it possible to determine to what extent these requirements must be changed to encompass vehicles subjected to aerodynamic heating.

3.2 Basic Areas of Structural Integrity

There appear to be three basic areas of structural integrity — "strength", "stiffness", and "life". "Strength" is a measure of the airframes ability to withstand a given load distribution without rupture or excessive yielding. To be rigorous, the time history of the load distribution should be included in this definition, as the amount of load that a structure can withstand is also a function of the dynamic response of the structure to the time history of the loading. However, as a detailed specification of the rate of load application and the mass and stiffness properties of the structure is necessary to compute the strength reserve of a structure under dynamic loading, general design specifications simply call out static strength requirements in the form of load factors for various types of aircraft.

"Stiffness" is a measure of the airframe's ability to withstand load distributions where the magnitude of the applied load is a function of the deformation of the airframe. The classic example of inadequate stiffness in the static sense is the static instability known as wing torsional divergence. The wing deflects torsionally increasing the airload which in turn increases the torsional deflection of the wing until failure results. The classic example of inadequate stiffness in the dynamic sense is the dynamic instability known as flutter, where the mass and stiffness of the wing and the flight velocity are such that periodic self-excited motions of a divergent nature are established with resultant structural failure. As the elasticity of the lifting surface plays an essential role in failure of the airframe due to phenomenon such as divergence or flutter, structural inadequacies of this type are classified as stiffness inadequacies. Present general structural design specifications call out stiffness requirements in terms of flutter and divergence speeds with appropriate safety factor margins.

"Life" is a measure of the number of operational flight hours that an aircraft can perform without failure due to cumulative damage phenomenon such as fatigue.

In general, if the structure is adequate with respect to strength, stiffness and life, the overall airframe level of structural integrity is sufficient.

3.3 Modes of Failure Associated with the Basic Areas of Structural Integrity

There are a number of ways in which a structure could prove inadequate for each of the three basic areas of structural integrity. Each one of these number of ways will be referred to as a "mode of failure". In order to avoid getting involved in any semantical circumlocutions as to what constitutes an all inclusive definition of a mode of failure, each mode will be defined in an "ad hoc" manner. For example, fatigue is a mode of failure; flutter also is a mode of failure. Consider now the modes of failure associated with strength, stiffness and life for the case of an aerodynamically heated structure.

3.3.1 Strength

For an unheated aircraft the structure can fail due to static strength inadequacy in only one way or mode. That is, if the external aerodynamic loading applied to the structure is such that the yield or ultimate strength of the structure is exceeded. Due to the mechanical properties of the materials used in the construction of modern airframes, sufficient ultimate strength generally automatically implies sufficient yield strength. In the case of the aerodynamically heated airframe the strength mode of failure is essentially the same, but the external load level that the structure can withstand is influenced by two phenomenon. These are (1) reduction of the ultimate (yield) strength and Young's modulus of the material with temperature and (2) thermal stresses due to temperature gradients within the structure. The amount of residual static strength inherent in a given structure (i. e., room temperature strength minus strength loss due to reduction in material properties and thermal stresses) is determined by (1) and (2). The structure fails when the external loading exceeds that which can be accommodated by the residual structural static strength. It should be noted that the material properties are not only a function of the structural temperature distribution but of the time at temperature as well. Hence, the material properties are a function of both time and temperature.

3.3.2 Stiffness

For a cold airframe the two most important stiffness design modes of failure are flutter and divergence. In the case of the heated airframe these will again be of primary design importance, with the flutter and divergence design conditions now being functions of the change in material properties with temperature and time, the thermal stress distribution in the structure, and in some cases skin buckling due to combined aerodynamic and thermal loadings.

3.3.3 Life

The only recognized life mode of failure associated with a cold airframe is fatigue. In the case of the heated structure fatigue remains important, and the new mode of failure known as creep is introduced into the design picture. Both the elevated temperature fatigue and creep life of a given structure are a function of the temperature and load time histories experienced by the airframe.

3.4 Probability of Failure of the Airframe

The previous section identifies the various modes of failure associated with strength, stiffness and life of a heated airframe. Before any evaluation as to the relative importance of these modes of failure could be accomplished it would be necessary to determine the contribution of each mode of failure to the overall failure rate or probability of failure of the airframe. Unfortunately, although suitable mathematical techniques are available, there is at present insufficient data on the failure rate associated with a particular mode to determine its contribution to the probability of failure of the airframe. As is pointed out in reference 1, aircraft structures in service are generally exposed to highly complex combinations of environment and loading, and as a result, it is not possible to assign appropriate relative weights to the contribution of strength, stiffness and life to the structural integrity of the airframe. Furthermore, in many cases the various modes of failure associated with stiffness, strength and life are coupled, i.e., they mutually influence each other. In spite of this inability to design structures quantitatively from the start including all failure phenomenon, it is felt (Refs. 2 and 3) that, as in the case of cold airframes, heated airframes will be designed from static strength considerations, with stiffness and life characteristics being checked after the initial design has been completed.

Although it is impossible to apply the probability of failure concept to an all inclusive, coupled treatment of the contribution of each mode of failure to the level of structural integrity of the airframe, it is still a useful discipline in the selection of critical design conditions for a given mode of failure. Specific examples of the application of this technique are given in Chapter IV and Appendix B of this report; so it is essential that the philosophy underlying the application of the probability of failure concept to structural design criteria be understood. This can best be accomplished by considering the following discussion of the gust loading of an airframe.

If the calculation of load due to gusts is considered a statistical process (which it in reality is), the selection of one maximum gust velocity as a design criterion cannot result in an aircraft which is absolutely certain of survival. Since gust velocities are random, there is some probability that there is a gust more severe than the one chosen for design. Thus, there is some chance of failure, as a result of encountering this more severe gust, for any airplane designed by a specified maximum gust velocity. If it were possible to predict the statistical properties of the gust loads that any particular airplane is likely to encounter, then it would be possible to calculate the probability that the aircraft would fail in a specified time interval. If the airplane is designed for a fifty foot per second gust then this design could be represented as a specified probability of failure during the life of the airplane. Furthermore, as there is a one to one correspondence between the design gust velocity and a probability of failure, then design criterion could be stated as a probability of failure rather than a specified maximum design gust velocity. Thus, if a statistical (probability of failure) design criterion were used, it would lead to the same design loads as the corresponding design gust velocity condition that is presently being used. It is simply another method of stating the same requirement.

The following chapter discusses the selection of critical design conditions for heated airframes; wherever possible within the probability of failure framework.

CHAPTER IV

SELECTION OF STRUCTURAL DESIGN CONDITIONS

4.1 General Considerations

The problem of selecting critical structural design conditions for airframes subjected to aerodynamic heating is a difficult one. The existing criteria will not be entirely adequate in this regard. To illustrate this, consider the three basic areas of structural integrity discussed in Chapter III, i.e., strength, stiffness and life (creep and fatigue). For airframes not subjected to aerodynamic heating it is possible to select certain discrete isolated load factors, altitudes, and velocities to use as design conditions for structural strength and stiffness. As mission requirements are not rigorously defined, the designer must consider many combinations of these three parameters before he is confident that the most severe combination has been selected as a design condition. It would be much better if the designer had statistical information indicating probabilities of occurrence of load factor level as a function of altitude and velocity for the design under consideration. However, in lieu of such information it is still possible for the designer to choose adequate design conditions by exercising experience and judgement. The important fact is that for aircraft not subjected to aerodynamic heating it is possible to select and utilize discrete isolated combinations of load factor, altitude, and velocity as design conditions regardless of the degree of rationality of the selection process. This is because, for strength and stiffness design, the history of the airframe up to the point of application of the critical load is not important. The aeroelastic instability speeds and the percentage of yield or ultimate load applied to the structure are strictly a function of the load factor, velocity and altitude experienced by the structure at a given point on the flight path — not of the previous history of these three parameters. (This statement neglects dynamic

alleviation effects on the allowable loads in the structure). Thus, for the unheated airframe the only basic area of structural integrity that is a function of the time history of the mission is that of cumulative damage. For fatigue design it is necessary to select a fatigue load spectrum, and this spectrum is a function of the mission profile of the airframe. As statistical information on probability of magnitude and order of application of load level is not as yet available the designer is forced to select (or the procuring agency forced to specify) a load spectrum somewhat arbitrarily selected. This, coupled with a lack of knowledge of the basic mechanism of fatigue, has led to some unsuccessful fatigue designs.

For airframes subjected to aerodynamic heating it is not possible to select certain discrete isolated load factors, altitudes and velocities for use as a design condition. Both the strength and stiffness required of the airframe at a given point in the mission are a function not only of the instantaneous application of load factor (in the case of strength) and the velocity and altitude (in the case of stiffness), but also of the past history of altitude and velocity. Both the yield and ultimate strength are functions of the temperature distribution in the structure, which is a function not only of the adiabatic wall or equilibrium temperature time history, but of the structural details as well. These effects are thoroughly discussed in, for example, references 4, 5 and 6. Reference 6 is especially interesting and applicable in that plastic effects have been considered. The torsional stiffness of a wing section is also a function of the temperature distribution in the wing; this temperature distribution again being a function of the airplane altitude and velocity time histories and the details of the structure. The reduction of torsional stiffness as a function of temperature distribution for wings of various cross-sectional shape and structural configurations is discussed in references 7, 8 and 9. Reference 9 discusses the effects of different materials of construction. Thus, it is seen that present practice of selecting isolated values of load factor, altitude and velocity as design conditions with no consideration of the previous history of these three parameters is inadequate in the case of aerodynamically heated aircraft. Instead, the entire mission or set of missions to be performed by the flight vehicle must be analyzed.

4.2 Mission Analysis

4.2.1 Definition of Mission

To determine the past history of a flight vehicle up to the time that, for example, a critical maneuver is to be executed requires that the designer consider all the possible ways of reaching this point. The difficulty of this task is a function of the degree to which the mission requirements of the flight vehicle are delineated. For a missile with a programmed mission profile this would be a relatively simple task as the velocity-altitude time history is defined (at least in the root mean square sense). For manned aircraft, however, complexities are introduced because of the variety of mission profiles to be experienced. Several authors (Refs. 10, 11 and 12) have suggested the application of statistical analysis to this problem. The approach of reference 12 appears to be the most general, and as such is discussed below.

A mission is defined as a combination of the time histories of three variables:

(1) $v = v(t)$ ~ vehicle velocity

(2) $h = h(t)$ ~ vehicle altitude

(3) $n = n(t)$ ~ vehicle normal load factor.

Thus, a mission "M_i" is denoted in the functional form as

$$M_i(t) = M_i\left[v_i(t); h_i(t), n_i(t)\right] \tag{4.1}$$

A typical mission time profile is illustrated in Figure 4.1. Each flight vehicle is assumed to be designed for one or several given missions. The validity of this assumption is indicated by the increasing trend toward the design of airframes for specific missions.

Although it seems reasonable to assume that a set of missions can be defined for a given flight vehicle configuration, and that a set of variables $v(t)$, $h(t)$, and $n(t)$ can be defined for each mission which would permit the successful completion of the mission, variations about these predetermined values can be expected due to, for example, gust, pilot or guidance system inputs. This concept can be stated symbolically as follows. For each mission M_i an associated set of values

$$v_i(t) = v_{o_i}(t) + \delta v_i(t)$$

$$h_i(t) = h_{o_i}(t) + \delta h_i(t)$$

$$n_i(t) = n_{o_i}(t) + \delta n_i(t) \tag{4.2}$$

exist which define the mission desired. v_{o_i}, h_{o_i} and n_{o_i} are the predetermined values of the velocity, altitude and load factor required to accomplish the mission; δv_i, δh_i and δn_i are the variations of these variables during the fulfillment of the mission. Assume these delta variations to be mutually independent, continuous random variables of normal distribution with standard deviation σ and zero mean value. The standard deviations will, for each of the three variables, be a function of the mission M_i and the time t as time increases from zero to the complete mission flight time $t = T_{M_i}$. The numerical values of the velocity, altitude and load factor standard deviations $\sigma_v(M_i, t)$, $\sigma_h(M_i, t)$ and $\sigma_n(M_i, t)$ should be based on simulation studies and the statistical flight data associated with each type of flight vehicle. Thus, the variables $v_i(t)$, $h_i(t)$ and $n_i(t)$ are normally distributed with standard deviations $\sigma_v(M_i t)$, $\sigma_h(M_i, t)$ and $\sigma_n(M_i, t)$, and mean values $v_{o_i}(t)$, $h_{o_i}(t)$, and $n_{o_i}(t)$.

It is assumed that a finite number of discrete missions exist and that an "a priori" probability of occurrence of each mission is defined such that the sum of these probabilities over all missions is one. Thus, for

each mission M_i characterized by $v_i(t)$, $h_i(t)$ and $n_i(t)$ there is an associated probability of occurrence $P(M_i)$ such that $\sum_{i=1}^{m} P(M_i) = 1$ where m is the number of missions.

4.2.2 Structural Temperature

In order to evaluate the adiabatic wall temperature T_{aw_i} at any time t, during a mission M_i, only the instantaneous values of the altitude and velocity must be known. However, to evaluate the temperature at a point in the structure at any time t_1 during the mission M_i, the time history of $T_{aw_i}(t)$ prior to the time t_1 must be known. This implies that the time history of the velocity $v_i(t)$ and the altitude $h_i(t)$ during the time range $0 \leq t \leq t_1$ must be known, and as $v_i(t)$ and $h_i(t)$ are random variables about the predictable mean values $v_{o_i}(t)$ and $h_{o_i}(t)$, to rigorously calculate the structural temperature at time t_1 would require defining the random behavior of the time histories of $v_i(t)$ and $h_i(t)$. However, if it is assumed that any variation from the mean flight path [i.e., $\delta v_i(t)$ and $\delta h_i(t)$] is such that the airframe returns to the mean flight path after a short time, and if such deviations are not of a large magnitude, the variation will have little effect on the structural temperature. This is because the time constants associated with the diffusivity of heat through the structure are significantly greater than that of, for example, a correcting, noncritical maneuver of a gust. Thus, it is assumed that the structural temperature at a given time during a given mission M_i is a function only of the prescribed mission [i.e., $v_{o_i}(t)$ and $h_{o_i}(t)$; $n_{o_i}(t)$ does not influence the temperature in the structure (Ref. 2)], and not of the random variation in altitude and velocity about their prescribed values. This assumption is justified in Appendix A where the magnitude of the time constant associated with thermal diffusivity is discussed. Hence, the structural temperature is associated in a one to one correspondence with a time during a given mission. In addition the structural configuration must be known before the temperature at any point in the structure can be determined.

4.2.3 Load and Temperature Combinations

Once the random characteristics of the load factor variation for each mission profile have been determined, it is possible to calculate the probabilities of occurrence of a given load factor. For example, the probability of the load factor n exceeding a given value n_k during an entire mission M_i is denoted in standard form by $P(n > n_k/M_i)$ and is defined as

$$P\left(\frac{n > n_k}{M_i}\right) = \frac{1}{\sqrt{2\pi}} \int_{n_k}^{\infty} \int_{0}^{T_{M_i}} \frac{1}{\sigma_n(t, M_i)} e^{-\frac{1}{2}\left[\frac{n - n_{oi}(t)}{\sigma_n(M_i, t)}\right]^2} dt\, dn \tag{4.3}$$

where T_{M_i} is the time duration of the mission M_i. This probability distribution gives the designer a means of evaluating the relative load factor severity of each mission, and as such would be a useful tool in the design of airframes not subjected to aerodynamic heating, (as time is integrated out, the structural temperature associated with each time value is obliterated). The most stringent mission can be selected, and the design load factor selected such that the probability of the airframe being subjected to such a load factor is sufficiently low. As the probability of occurrence of each mission is assumed to be known, it is possible to arrive at a probability of exceeding a given load factor taking into consideration the entire ensemble of mission. This probability is simply expressed as

$$P(n > n_k) = \sum_{i=1}^{m} P(n > n_k,\ M = M_i) \tag{4.4}$$

where m is the number of missions and $P(n > n_k,\ M = M_i)$ is the joint probability of the combined event $n > n_k$ and $M = M_i$. This can be shown to be (Ref. 13) equivalent to the expression

$$P(n > n_k) = \sum_{i=1}^{m} P\left(\frac{n > n_k}{M_i}\right) p(M = M_i) \qquad (4.5)$$

where $P(M = M_i)$ is assumed known and $P(n > n_k/M_i)$ is determined by evaluating Eq. (4.3) for each mission. Thus, it is possible to calculate the probability of failure associated with each load factor n_k, and the design load factor can be selected such that the probability of airframe failure is sufficiently low.

A more basic distribution to utilize in the structural design of airframes subjected to aerodynamic heating is the probability of the load factor exceeding a given value n_k at a time t during a mission M_i. This distribution is denoted in standard form as $P(n > n_k/M_i, t)$ and is defined as

$$P\left(\frac{n > n_k}{M_i, t}\right) = \frac{1}{\sqrt{2\pi}} \int_{n_k}^{\infty} \frac{1}{\sigma_n(t, M_i)} e^{-\frac{1}{2}\left[\frac{n - n_{oi}(t)}{\sigma_n(M_i, t)}\right]^2} dn \qquad (4.6)$$

This probability distribution can serve as a basic tool in establishing design criteria for heated airframes, as it determines the probability of exceeding a given load factor in concurrence with a unique structural temperature distribution determined by M_i and t.

4.3 Possible Design Criteria for Heated Structures

4.3.1 Preliminary Discussion

As is pointed out in Section 4.1, it is necessary to have a knowledge of the altitude and velocity time histories to be experienced by a heated airframe before the strength, stiffness and life capabilities of the airframe can be determined and critical design conditions selected. Section 4.2 discusses a method of specifying the load factor in a statistical sense. It is felt that such an approach is warranted as the trend is definitely toward well defined flight profiles, and information on gust and maneuver statistics is becoming increasingly available. Mission optimization is receiving more attention, and as is pointed out in reference 3 the flight profile for aircraft in the regime where aerodynamic heating effects are important is likely to be simple and well defined. For example, if the aircraft is air breathing, by virtue of its high Mach number it will employ focussed shock intakes in some form, and will achieve its lift to drag ratio by shock cancellation techniques. As both of these procedures are effective over a small Mach number tolerance, sustained flight at such Mach numbers certainly implies a long range function for which flight programing will be essential, since off design conditions give low air miles for a given fuel consumption. As a result, the climb and acceleration phases will themselves be closely controlled. As another example, in the case of a missile or high performance fighter designed to intercept a high speed bomber, the accuracy with which the interception vehicles guidance system can predict the position of the bomber will determine the load factor that the intercepting vehicle will have to experience to reach a point where it can destroy the bomber. As methods of interception prediction are statistical in nature, the load factor to be experienced by the vehicle is best treated in a statistical sense. An example of this type of problem is discussed in reference 11. An evaluation or analytical extension of such methods is beyond the confines of this report; it suffices to say that in the future flight vehicles will be structurally designed on the basis of a system analysis which accounts for such items as

optimization of number of vehicles to be utilized to perform a given mission, guidonce and fire control charocteristics, etc.

The following sections develop structural design criteria based on the statistical mission analysis of Section 4.2. Other more sophisticated probability anolyses could be used; the method chosen adequotely demonstrates the principles involved. The actuol criteria per se need not specify a particular statistical method just as the present criteria does not specify a particular stress analysis method.

4.3.2 Strength Design Limit Load Factor Requirements

At present design limit loads ore specified for eoch class of aircraft. That is, there is a different load factor velocity envelope for each class of aircraft. It is significant to note that the number of classes of aircraft is constantly increasing; each closs requiring a new load factor velocity envelope based on its desired mission capabilities. This illustrates the trend toward designing a specific vehicle to perform a specific task. This trend has an important implication. Although the load factor-velocity envelope is still specified, the particular missions to be performed within the envelope are much better defined than they were in the past. This, it is felt, makes it possible to perform a mission analysis based on probability considerations for each class of aircraft to determine what the magnitude of thermal load to be superimposed upon the load factor should be and at what altitude and velocity it should be applied. There are many ways to do this, and the approach to the problem given below is intended to be strictly illustrative.

It is assumed that enough information is known about the mission requirements of the aircraft and the possible load factors to be encountered that an expression such as Eq. (4.6) for the probability of occurrence of a given load factor can be established. It is not necessary that such an expression be entirely rigorous os it is not going to serve as a limit load factor criterion itself, but simply to indicate where a thermal load might logically

be superposed upon the limit load factor. The limit load factor as it is presently specified will be maintained. When mission profiles become statistically defined with a sufficient amount of rigor, it will be possible to eliminate the envelope type limit load entirely and select both the load and thermal critical design conditions from a mission analysis. In the meantime, it is best to maintain the current limit load envelope and limit the use of the as yet unproven statistical approaches to determining where the thermal effects should be superposed for purposes of design. The following procedure could be utilized.

1. Divide each mission time history (e.g., Figure 4.1) into several time increments. For each time value evaluate Eq. (4.6) or its equivalent for a range of load factors n_k. This results in a family of curves, one curve for each time value. A typical family is illustrated in Figure 4.2, where to avoid confusion only the curves for two values of time t_1 and t_2 are shown. For each mission there will be a corresponding family of curves. Note that as the standard deviation $\sigma_n(t, M_i)$ and mean $n_{o_i}(t)$ are functions of time, both the shape and mean value of the probability distribution curve change with time. In all cases the mean $n_{o_i}(t)$ has a probability of 0.5. One of the important characteristics of the normal distribution is that it is completely determined by its mean and standard deviation.

2. Assume a structural configuration. This is necessary at this point as it is impossible to calculate thermal stresses and strength modulus reductions without a detailed knowledge of the structure.

3. As the mission profiles are known, and as was discussed earlier, it is assumed that small variations about the planned mission velocity-altitude time histories do not affect the structural temperature distribution, the temperature distribution at a critical point in the structure can be determined for each of the time values selected in (1) for each mission. The equilibrium temperature chart presented in Appendix C can be used in this regard.

4. Using the temperature distributions obtained in (3) calculate the thermal stress distribution and the amount of reduction in strength modulus due to these temperature distributions.

5. Utilizing the thermal stresses and strength reductions calculated in (4), determine the residual strength of the structure. This is simply the room temperature strength minus strength loss due to thermal stresses and reduction in strength modulus due to temperature, and is a measure of the amount of external loading that can be applied to the structure.

6. From the residual strength determined in step (5) calculate the various residual load factors n_k that the structure can withstand at each time value for each mission.

7. Using the plots corresponding to Figure 4.2 obtain the probability of the airframe being subjected to a load factor in excess of each residual n_k for the various time values for each mission. From this array (all times and missions) select the most critical condition, i.e., the time and mission where the probability of the residual load factor being exceeded is the greatest.

8. As the mission and time for the critical loading condition established in (7) are now known, these two parameters define the critical altitude and velocity and also the time history of the altitude and velocity prior to this critical point. This is precisely the desired information. The strength design condition is now that the structure must be able to withstand the limit load as specified by the V - n diagram of MIL-A-8860(ASG) at the established critical velocity and altitude taking into account the thermal stresses and strength modulus reduction resulting from the temperature distribution associated with the altitude and velocity time history prior to reaching the critical point. If the structure selected in step (2) is inadequate (i. e., either too strong or not strong enough), the procedure outlined above must be repeated. It is unfortunate that the feedback between structural configuration and strength degradation due to elevated temperature exists; however, as the original mission requirements, in conjunction with past design experience, will in most cases define the structure within reasonable limits before its integrity is analytically demonstrated at the critical thermal design condition, this problem should not be serious.

Thus, a workable limit load design criteria for heated structures has been established. The method has the following advantages:

1. The present design specification limit load requirements are maintained. The probability of load factor occurrence concept is used simply to determine where the thermal load should be superposed on the nonheated structure limit load. Hence, the past operational

experience that is reflected in the limit loads called out in MIL-A-8860(ASG) is essentially transferred to the heated airframe. This is desirable.

2. The method of selecting the altitude and velocity at which to apply the thermal load, and of determining the past history of altitude and velocity to use in calculating the thermal load magnitude is as rational as is possible at present. It selects the critical values of altitude, velocity and thermal penalty from probability of failure considerations which take into account both the load factor due to aerodynamic and inertia forces and the thermal effects. As such, it is less stringent and more realistic than superposing maximum thermal effects on maximum load factor. This is shown in Figure 4.2 for the simple case of one mission divided into two time increments t_1 and t_2. From the structural temperature distributions calculated for times t_1 and t_2 it is assumed, for purposes of illustration, that the residual strength at t_1 is greater than that at t_2, i.e., the structure at t_1 can withstand a load factor n_1 which is in excess of the load factor n_2 that can be withstood at t_2. The time t_2 then represents the most critical thermal condition, i.e., the structure can accommodate less aerodynamic and inertia load at t_2 than it can at t_1. Thus, if a critical design condition were to be determined from thermal considerations alone, the thermal stresses, etc. associated with time t_2 would be selected for design purposes. However, if both external load factor and temperature are included in the critical design condition selection process, Figure 4.2 indicates the probability of exceeding the allowable (residual) load factor n_1

is greater than that of exceeding the load factor n_2 even though n_1 is larger than n_2; hence, time t_1 is in actuality the critical design condition. It should be re-emphasized that the given example is a gross oversimplification of an actual case where a large number of missions and time increments must be considered. The concepts involved, however, appear to be physically sound; for example, a fighter with a programmed mission may experience severe strength reduction due to temperature at a particular time during the mission, but the operational requirement may be such that the necessity for maneuvering at this time is doubtful; hence, the probability of exceeding the allowable load is low. This corresponds to the time t_2 in the above example. At another time during the mission the thermal effects may not be so severe, (i.e., the structure can accommodate a comparatively greater aerodynamic and inertia load) but the chance of having to maneuver (or encountering severe atmospheric turbulence) is high enough such that the probability of exceeding the allowable load factor in this case is higher than before. This is obviously the most critical of the two conditions, and corresponds to the time t_1 in the above example.

3. The method is quite flexible. The best possible method of predicting the critical values of the pertinent design parameters within the probability of failure framework can be utilized; thus as superior and more rigorous methods become available they can be used. The design specification should not specify a particular methodology in this regard. The methods used by the contractor should be approved by the procuring agency, but the selection of method(s) should be the contractors

responsibility. The method chosen for illustration is based on the assumption of a simple normal distribution of load factor. This assumption may be valid in some instances and invalid in others. The design criteria should simply state that the structure will be able to withstand the presently specified design limit load at a velocity and altitude where the probability of the load factor exceeding that which can be sustained by the structure is a maximum. In many cases it will not be necessary for the contractor to undertake elaborate probability analyses. For example, if an attack aircraft is expected to maneuver after a dive during which a violent change in Mach number and altitude occurs, this will probably be the critical condition as high aerodynamic and inertia forces must be sustained by a structure experiencing maximal thermal effects.

The main disadvantage of the proposed method is that it requires a detailed knowledge of the structure. However, as the prediction of any of the several structural effects of aerodynamic heating is predicated on a detailed knowledge of the structure, it is difficult to see how this unfortunate situation can be obviated.

If it is felt that another revision of aircraft structural design criteria specification is in order before any probability of load level occurrence requirements be specified as part of the design requirements, several authors have suggested approaches to the problem of specifying design loads that include heating effects. References 14, 15 and 16 are examples. It should be emphasized that there is a modicum of rational justification for these methods; they are based largely on the experience and judgement of the individuals concerned. Of these several procedures, that of reference 16 appears to be well thought out, has the advantage of being simple, and as such is presented below.

1. Because of aerodynamic heating effects on supersonic aircraft it is necessary to specify design conditions in a more elaborate manner than has hitherto been the case, by including those elements of a particular flight plan of an aircraft which involve accelerations or decelerations from one speed to another. It is understood that the optimum climb paths for minimum fuel usage of supersonic aircraft now being considered may need to be modified to keep the speed at a reasonable margin below the design diving speed. As the Mach number during the climb phase is subsonic, it is not considered that any special aerodynamic heating problem exists during this phase. It is assumed that the aircraft will climb to some height, e.g., 30,000 feet or 40,000 feet before leveling off and accelerating through $M = 1.0$ to supersonic speeds.

2. Similarly during the descent phase it may be necessary to impose pilot's limitations, keeping the speed to some constant fraction of the design diving speed. It is assumed that during the descent, the deceleration from supersonic to subsonic speeds will be done in level flight at cruise altitudes. No aerodynamic heating problem appears to arise during the descent phase.

3. So far as aerodynamic heating is concerned, the stages of the flight path which have to be considered are

   a. Acceleration from subsonic speed to supersonic speed.

   b. Cruise at supersonic speed.

   c. Deceleration to subsonic speed.

The problem is to consider how these phases should be combined with the normal flight envelope cases. In the following it is assumed that V_{dive} will only be obtained

by accident as an overshoot beyond V_{cruise}. The following table summarizes suggestions in this regard.

Case	Speed to be considered	Flight loads to be combined with thermal loads
i. Acceleration from subsonic speed to supersonic cruise speed.	From M = 0.8 or 0.9 to V_{cruise}.	2/3 flight envelope loads or 2/3 standard gust loads.
ii. Same, but overshooting to V_{dive}.	V_{dive}	1/2 flight envelope loads. 1/2 standard gust loads.
iii. Supersonic Cruise	V_{cruise}	Flight envelope or standard gust loads.
iv. Decelerated flight to subsonic speed.	From V_{cruise} to M = 0.8 or 0.9	2/3 flight envelope loads or 2/3 standard gust loads.

4. In case (i) the flight envelope loads and gust loads have been reduced to two-thirds of their normal value on the assumption that full flight envelope or full gust loads may be considered as rare events not to be experienced on every flight. In case (ii) the flight envelope and gust loads have been halved on the assumption that accidental overshoot to V_{dive} is a rare event. Case (iii) is the saturated condition — full flight envelope or gust factors being combined with thermal stresses. In case (iv) decelerated flight is considered. The argument supporting case (i) is valid in this case.

To conclude this section on limit load factors it should be noted that a method of utilizing a probability of failure approach in the case of heated airframe gust loading has been developed in reference 17. As this method has received little attention to date, and is felt to be a pioneering effort in this regard, it is presented in Appendix B. A similar development is presented in reference 18 for the gust loading of an unheated airframe. Essentially, what is done in both cases is to restate the present discrete design gust velocity requirement in terms of the probability of exceeding the gust design limit load factor. As this probability of failure is based on the discrete gust requirement — which has been in service for many years — it is assumed to be acceptable. This acceptable probability of failure is then applied to a new design (possibly subjected to aerodynamic heating) to determine the design gust load factor for the new airframe. Thus, a correspondence is established between the satisfactory, but somewhat arbitrary gust criteria of the past, and the untested but more rational statistical approach.

4.3.3 Stiffness Design Requirements

In the case of aircraft structures subjected to aerodynamic heating, the wing torsional and bending rigidities are a function of the temperature distribution in the wing. Section 4.2.2 of this report indicates that this temperature distribution at a given time is a function of the time history of the mean altitude and velocity prior to the given time, and of the structural configuration, but not of the random variations of the altitude and velocity about their respective mean values. Thus, once the mission profiles are defined in the root mean square sense, the wing temperature distribution time history can be calculated for each mission. This determines the torsional and bending rigidities of the wing for each time during a given mission. As the altitude and velocity time histories are known for each mission, there is a velocity and altitude associated with each of the sets of values for the torsional and bending rigidites; hence the flutter and divergence speed for each time may be calculated. An example of this type of calculation is

given in reference 9, where the flutter and divergence speed margins are presented in Figures 9 and 5 respectively of reference 9. It should be noted that both the flutter and divergence speeds are fictitious quantities, since they are speeds computed from the stiffness of the wing at a particular instant in its flight mission and not from the stiffness corresponding to flight at the actual flutter or divergence speed. Such a fictitious speed is useful in indicating the flutter or divergence margin, and a point of intersection of a flutter or divergence curve with the flight mission curve would correspond to a real flutter or divergence point respectively.

From the criteria selection standpoint, the pertinent factor is that for a given aircraft the flutter and divergence speed margins are not only a function of the velocity and altitude at a given point on the flight path but also of the altitude and velocity time histories prior to this point. Thus, the envelope type design criteria as illustrated in Figure 1(a) of MIL-A-8870(ASG) will either have to be replaced or modified to include the change in rigidity associated with the several ways in which a given airframe is expected to reach a given altitude-Mach number combination. Eventually, the envelope type flutter and divergence criteria should be replaced by a mission type analysis such as that discussed above, and illustrated in Figure 4.3. The lower curve in Figure 4.3 gives the value of the rigidity parameter for the actual wing as a function of Mach number for a given mission profile. Essentially, it is a time history of the rigidity parameter for the mission defined in terms of altitude and Mach number. The upper curve gives the value of the rigidity parameter required for occurrence of bending-torsion wing flutter. The difference between the two curves represents the flutter margin. Again, note that the flutter margin is a fictitious quantity as it is computed from the stiffness of the wing at a particular instant during the mission, and not from the stiffness associated with the actual flutter speed.

In addition to the above, the following recommendations are suggested.

1. That the 1.15 V_e margin as presently specified in MIL-A-8870(ASG) should be maintained. The most common significant deviation from the mission flight path will still be in velocity, and, as is shown previously, due to the slow response of the structural temperature to a Mach number change, this velocity overshoot will not significantly change the rigidity of the lifting surface; hence the stiffness values computed for a given V_e will be valid for a 1.15 V_e overshoot.

2. That paragraph 3.1.1 of MIL-A-8870(ASG) be modified to read, for example, as follows: "There shall be no flutter, buzz, or other related dynamic instabilities or divergence of the airplane or its components at all speeds up to 1.15 V_L for all design ranges of altitudes, maneuvers, and loading conditions. To assure safety, it shall be shown by analytical or experimental data or both that an increase of fifteen percent in equivalent airspeed at all points on the altitude Mach number time history for each mission to be performed by the airplane will not result in flutter or divergence. The values of rigidity used to calculate the flutter or divergence boundary at a given point during a mission shall include any change in rigidity due to thermal conditions associated with the point. In addition the damping . . . "

   It should be noted that the above possible proposed requirement includes the existing requirements specified in paragraph 3.1.1 of MIL-A-8870(ASG). Also, the phrase "change in rigidity due to thermal

conditions" should be interpreted to include the effects of thermal elastic and plastic buckling on rigidity as well as the more obvious effects of thermal stress and material property degradation.

4.3.4 Fatigue and Creep Design Requirements

A detailed knowledge of the number of cycles at various load levels and temperatures is necessary to evaluate the fatigue and creep characteristics of a given structure. In addition, a knowledge of the strain rate at various temperature and load levels is necessary, as an airframe subjected to transient aerodynamic heating will experience thermal strain in addition to thermal stress. In the elastic case there is no difference in fatigue under strain cycling and fatigue under stress cycling. In the high temperature range, where creep deformation plays an important role, a constant strain will mean a gradual relaxation in stress. Hence, there is an important difference between strain cycling and stress cycling, and a knoweldge of when each occurs during the operation of the flight vehicle is essential. This implies a detailed knowledge of the mission profiles for the vehicle. The picture is further complicated by the fact that fatigue and creep laws are rudimentary and inadequate, and that fatigue-creep interaction effects are virtually unknown.

In the case of the elevated temperature fatigue design criterion, a stress-temperature spectrum must be defined in much the same way as the load spectrum is defined in MIL-A-8866(ASG) for room temperature fatigue. The temperatures associated with each load level will have to be selected on the basis of mission analyses. A possible method of accomplishing this is outlined in reference 12.

The establishment of a detailed creep design requirement would be difficult due to the complexities discussed above. Also, the selection of a criterion to be applied in determining when the structure must be discarded

as a consequence of creep distortions is a matter to be decided in each individual case. Hence, it is recommended that the structural design specification simply call out a requirement to the effect that the permanent deformation of the structure shall not be so large as to render the airframe unsatisfactory from an aerodynamic or control point of view.

CHAPTER V

SAFETY FACTOR CONSIDERATIONS

5.1 Introduction

The previous chapter discusses the problem of selecting critical conditions to use in the structural design of airframes subjected to aerodynamic heating. Possible structural design requirements are suggested for each of the three basic areas of structural integrity — strength, stiffness and life. The purpose of this chapter is to discuss the nature and magnitude of possible safety factors to be applied to these structural design requirements. Before this can be accomplished, however, it is necessary to investigate the effectiveness of the safety factor in providing an adequate level of structural integrity (i.e., a sufficiently low probability of failure) in the case of existing nonheated airframes. Using this information, in conjunction with a knowledge of the effects of aerodynamic heating on airframe strength, stiffness and life, it is possible to determine the applicability of the safety factor concept to heated airframes.

The following section of this chapter discusses the safety factor concept as applied to extant airframes. The final section discusses the safety factor as applied to heated airframes, and makes several recommendations in that regard.

5.2 Safety Factors as Applied to Nonheated Airframes

The present structural design criterion specifies strength requirements by establishing a level of static strength using specified loading conditions (due to aerodynamic and inertia forces) called limit loads, which are supposed to occur about once in the life of the airframe. The ultimate load (at which

failure of the structure is permissible) is the limit load multiplied by a 1.5 factor of safety. This safety factor is intended to provide for unknown deficiencies in strength as well as for unpredictable incidents producing excessively severe loads. In essence, the safety factor is provided to help keep the probability of failure of the airframe below a tolerable level. The effectiveness of the safety factor in this regard must be established.

The 1.5 safety factor was originally proposed to allow for unknown factors such as stiffness and life as well as inadvertent strength deficiencies (Ref. 3). The modes of failure associated with stiffness and life were essentially unknown as such and not accounted for in the design procedure. Aircraft exhibited limited performance capabilities and were designed to accomplish a broad array of missions; hence, it was difficult to establish critical loading conditions upon which to base limit loads for strength design. As a result, limit loads were in general high, (particularly for fighters where ultimate positive load factors were based on human capabilities to withstand acceleration). Speeds were low enough to preclude any aeroelastic phenomenon (Ref. 2), and the 1.5 safety factor, in conjunction with the yield stress-limit load requirement, had the effect of restricting the stresses at load levels which were often reached in service (Ref. 19). This was a satisfactory, if inadvertent, method of preventing fatigue. In summary, for low performance aircraft, the 1.5 safety factor as applied to the limit load was satisfactory in preventing structural failure due to inadequate strength, stiffness or life characteristics. Stiffness and life design conditions were not required; hence, there were no safety factors associated with these phenomenon.

As airplane performance (particularly speed) increased, it became obvious that airframes could not be structurally designed on the basis of strength alone. It became necessary to include stiffness and life considerations in the design process (Refs. 2 and 3). The 1.5 safety factor on limit load continued to supply a sufficient level of static strength, but it no longer guaranteed satisfactory stiffness integrity or fatigue life (Ref. 19); hence design criteria

for stiffness and life were established. The stiffness design criteria incorporated its own safety factor on the design flutter and divergence speeds (see Chapter IV of this report) while fatigue, being a more subtle and less understood mode of failure, did not prove amenable to the application of a simple multiplying safety factor. By prescribing, for example, that the gust loading spectrum for fatigue shall include at least one percent of the airplane's life at sea level, equivalent "safety factors" are inherent in the fatigue specification. It is impossible at present to determine the effectiveness of such an indirect safety factor, as insufficient operational data is available for evaluation. However, it is certainly a rational requirement as a relatively small increase in the percentage of actual flight time at sea level can significantly reduce the fatigue life of an airframe.

In spite of the fact that the 1.5 safety factor was originally established in part to allow for unknown phenomena that are now more or less rigorously investigated, the full value of the 1.5 factor is maintained. The need for this is questionable, as the modes of failure associated with stiffness and life are now specifically included in the structural design process. Reference 19 suggests a method for determining the magnitude of the safety factor necessary to account for discrepancies between actual strength and required strength, i.e., the safety factor magnitude necessary to provide a sufficient level of structural strength integrity. Essentially what is done is, to determine the magnitude of safety factor needed to keep the probability of strength failure below a certain tolerable limit. This probability of failure depends on

(a) the probability of occurrence of various load levels, and

(b) the scatter of actual aircraft strength.

This may be expressed mathematically as

$$P = \int_0^\infty f_x \, a_x \, dx \qquad (5.1)$$

where

f_x = probability per flight hour of exceeding a load x

a_x = probability of aircraft strength being between x and $x + dx$

P = probability of strength failure per flight hour.

The probability distribution f_x is essentially a limit load spectrum while the frequency distribution a_x is a function of the various uncertainties with respect to airframe strength. These include such items as ignorance of allowable stresses, ignorance of actual material properties, imperfect simulation of applied loads in static test, manufacturing errors, deficient maintenance, etc. It is not necessary to formulate an all inclusive list for purposes of discussion.

By assuming a broad range of values for a_x and f_x, reference 19 shows that a safety factor of 1.2 would insure a sufficient level of strength integrity providing the present levels of accuracy of load specification and design strength attainment are maintained. This conclusion is in a sense corroborated in that operational experience accumulated by aircraft of various types indicates that there have been very few static failures in flight (Ref. 3). Also, a brief but revealing investigation of Navy airframe failure records indicates that there has been little, if any, increase in strength failure rate with increased aircraft performance.

Of course, it is impossible to state with assurance that an adequate level of structural integrity could be maintained by lowering the safety factor to 1.2. Even with stiffness and fatigue criterion written into the design specification, new methods of construction and new materials introduce design uncertainties that are impossible to predict. Also, there is a possibility that lowering the safety factor to 1.2 would have an adverse effect on the low cycle-high stress life of a high performance fighter. It is difficult to predict the cumulative damage due to this phenomenon. However, on the basis of the best predictions possible, the indications are that the

safety factor for nonheated airframes could be lowered to 1.2. The stiffness criteria calls out its own safety factor in defining aeroelastic instability margins, and fatigue analysis and tests are mandatory. This, coupled with careful design and test, certainly removes a large burden from the 1.5 safety factor as applied to the limit load.

It is important to note that a trend is discernible. That is, as airplane performance increases, less and less reliance can be placed on an overall safety factor applied to the limit load. Basically, the only rational function performed by the safety factor as applied to the limit load is to cover inadvertent strength inadequacies — it is an indirect, ineffective and irrational method of providing adequate stiffness and life structural integrity. Modes of failure associated with stiffness and life must be designed for directly, with their own margins or factors of safety. This trend in general continues for the case of airframes subjected to aerodynamic heating.

5.3 Safety Factors as Applied to Heated Airframes

Consider now the applicability of the safety factor concept in the structural design of heated airframes; in particular with respect to strength, stiffness and life design requirements.

5.3.1 Strength Design Safety Factors

Obviously some magnitude of safety factor must be utilized in the strength design of the heated structure. Manufacturing errors, deficient maintenance, etc. represent a design uncertainty — probably of the same order of magnitude as in the case of the nonheated airframe. The previous section of this chapter indicates that a safety factor of 1.2 applied to the limit load would insure adequate strength for a nonheated airframe. In the case of the heated airframe three questions must be answered. First, how many safety factors should be specified; second, where should the safety factor(s) be applied; and third, what magnitude of safety factor(s) should be

incorporated. The answers to these three questions are discussed below.

It is desirable that, as is the case with nonheated structures, a single numerical strength safety factor be specified. In the case of nonheated structures it is applied to the limit load, i.e., to the aerodynamic and inertia loading experienced by the airframe. This limit load safety factor is intended to cover not only inaccuracies in estimating the loading applied to the airframe, but also a plethora of other possible sources of inadequate strength. It is simply applied to the limit load as a matter of convenience as it would be too complex to specify individual safety factors for errors in limit load estimation, allowable strength, manufacturing, etc. This process of applying a single "lumped" safety factor has been successfully used for years; there is certainly no apparent reason why aerodynamic heating effects should suddenly invalidate the concept. Specification of a dual safety factor would unnecessarily complicate the design process, and could conceivably lead to confusion. In this regard it should be noted that several papers on the subject of design criteria for heated aircraft recommend that an additional safety factor be applied to the thermal effects (e.g., Ref. 16). It is not felt that this is justifiable for several reasons.

First, the probability of exceeding the design structural temperature distribution is much lower than that of exceeding the design limit load factor. The design limit load factor can be exceeded simply by encountering a large gust or performing a violent maneuver, but it is difficult to exceed the design temperature distribution as any sudden, unpredictable change in aircraft velocity or altitude (with subsequent return to a desired velocity and altitude) is not reflected as a change in structural temperature. This is due to the thermal time lag associated with the structure and is discussed in Chapter IV and Appendix A of this report. Thus, one of the main reasons for the specification of a safety factor (i.e., allowing for the structural consequences of an unpredictability in the environment of the aircraft) has been mitigated to a great degree.

Second, the primary strength effects of aerodynamic heating are thermal stresses and material properties degradation. Thermal stresses contribute to the total stress in the structure, while material properties degradation influences the structural allowables. At present (i. e., for nonheated structures) there is no additional safety factor specified for stress distribution and allowable strength, as all inadequacies with respect to these two phenomenon are covered by the 1.5 limit load safety factor. It would certainly be inconsistent to specify an additional safety factor to be applied, for example, to just the thermal portion of the stress distribution and not to the total stress distribution.

Third, the effectiveness of a safety factor applied to either the temperature distribution or the thermal stress distribution is doubtful. The application of a safety factor to the temperature distribution will not necessarily result in a more conservative value for the elastic thermal stresses. The importance of accurately determining the temperature distribution is discussed in reference 20, where it is shown that in some cases conservative temperature distributions (e. g., safety factored temperature distributions) yield <u>unconservative</u> thermal stresses. This clearly indicates the difficulty in applying a safety factor to a temperature distribution. Reference 21 indicates the importance of utilizing the correct method of calculating thermal stresses. The conventional approximate beam thermal stress analysis yields an erroneous result compared to a more sophisticated analysis. Multiplication of the stresses found by an inaccurate method by a safety factor would not necessarily be conservative in comparison with the unfactored exact results.

Now that the inadvisability of specifying an additional safety factor for thermal effects has been established an answer can be given to the question of where the recommended single numerical strength safety factor should be applied. The answer is quite simple. The safety factor is presently applied to the limit load; this practice should be continued in the case of airframes subjected to aerodynamic heating. The limit load safety factor has provided an adequate level of strength integrity in the past, and the authors see no reason why the advent of aerodynamic heating should negate this

effectiveness. Furthermore, as is pointed out above, the primary strength consequences of aerodynamic heating are thermal stresses and material properties reduction, which contribute to the total stress in the structure and the material allowables respectively. As inadequacies in these two design factors are at present covered by a limit load safety factor, it is certainly logical to assume that they could be adequately covered by a limit load safety factor in the case of heated airframes.

The only question remaining to be answered is that concerning the magnitude of the safety factor to be specified. Section 5.2 indicates that a 1.2 safety factor might be adequate for nonheated airframes. It is doubtful that this would be the case for heated airframes as several new uncertainties enter the design picture. First, the design allowables are a function of the temperature dependent material properties (Young's modulus, yield stress and ultimate stress) and are thus a function of the temperature distribution within the structure. This certainly decreases the accuracy with which the allowable stress at a point in the structure can be determined, as errors in temperature distribution are reflected as errors in allowable stress. Also, different regions of the same part will be at different temperatures, thus further complicating the allowable stress prediction. Second, one of the primary reasons for the adequate strength level of existing aircraft is the static test. Techniques and methods for testing nonheated structures have developed to a state where the operational strength adequacy of the structure can be accurately demonstrated in static test. This may not be the case for heated structures due to the difficulty in predicting the temperature distribution in the structure and then simultaneously simulating this temperature distribution and the airloads. In essence, aerodynamic heating effects will probably increase the scatter of actual aircraft strength, thus adversely affecting the probability distribution a_x of Eq. (5.1). Hence, a safety factor in excess of 1.2 will be required.

Reference 3 suggests that it should be possible to cover these additional uncertainties due to aerodynamic heating within a 1.5 safety factor. It is probable that this is true, particularly if a factor of 1.2 would be adequate for nonheated structures. Also, as is pointed out previously in this section, there is little need for the safety factor to allow for changes in structural temperature distribution due to unpredictable changes in aircraft velocity and altitude. Thus, with regard to thermal effects, the safety factor need only provide for the scatter in aircraft strength.

In view of the above, it is recommended that the 1.5 safety factor as applied to the limit load be maintained, and that no additional safety factors be incorporated in the strength design of heated airframes. The strength design requirement should be such that the structure must be able to withstand 1.5 times the limit load, taking into account the unfactored strength loss due to thermal effects. This is illustrated in Figure 5.1. The selection of critical thermal design conditions in this regard is discussed in Section 4.3.2.

5.3.2 Stiffness Design Safety Factor

The desirability of maintaining the 1.15 Ve safety margin in the stiffness design requirements is discussed in Section 4.3.3 of this report; further elaboration is not necessary.

5.3.3 Life Design Safety Factor

Thermal fatigue and creep are not amenable to the application of a simple multiplying safety factor. As is pointed out in Section 5.2, a correct approach to this problem has been initiated in the case of fatigue, where the one percent at sea level requirement has been established. Before similar requirements can be written for thermal fatigue and creep, thermal and load spectrums must be determined from future planned mission requirements to facilitate the selection of critical design spectrums. Particular emphasis should be placed on the low cycle-high stress thermal fatigue problem.

CHAPTER VI

CONCLUSIONS AND RECOMMENDATIONS

6.1 Introduction

Most of the conclusions and recommendations resulting from this study are stated where they were developed in the previous chapters of this report; hence, they will not be repeated here. However, several general conclusions, along with some additional recommendations not presented previously, are given below.

6.2 General Conclusions

1. In general it appears that airframes subject to aerodynamic heating can be designed with adequate strength and stiffness characteristics by using currently available methods of determining the effects of elevated temperature on strength and stiffness. The 1.5 limit load safety factor and the 1.15 aeroelastic speed margins should adequately cover respective inadequacies in this regard. In essence, it is felt that the strength and stiffness design of heated airframes will not present any major problems, particularly when load level statistics become available. In the future it may even be possible to reduce the 1.5 safety factor, although this is not recommended at present. This would be desirable as with increased performance each pound of structural weight must be justified, and a reduction in safety factor would significantly lower the structural weight. This is of particular importance in

missile design, where the percentage of gross weight allocated to the structure is lower than in the case of manned aircraft.

2. The problem of designing heated airframes with adequate life characteristics will be of paramount importance. Fatigue and creep design conditions must be determined and included in the design criteria. Until this is accomplished, it is difficult to visualize the procurement of heated vehicles with consistently adequate fatigue and creep characteristics. The procuring agencies should initiate efforts in this direction as soon as possible.

6.3 Additional Recommendations

1. Possible methods of demonstrating compliance with a design criteria should not be written into the design specification. The purpose of the design criteria is twofold; first, to force the designer to consider pertinent structural design problems, and second, to promulgate a design condition (i. e., what to design for) for each of these problems. Methods of demonstrating compliance should be chosen (or developed) and utilized in each case by the airframe manufacturer in a manner such that compliance with the specification is demonstrated to the satisfaction of the procuring agency.

2. Any new criteria should be evolved independently of any proposed or extant structural testing criteria. Design criteria should not be influenced by the limitations inherent in test criteria; in fact, test criteria should be evolved in the light of design criteria, not vice versa. In essence, allowing test criteria to influence design criteria would have a confining influence.

APPENDIX A

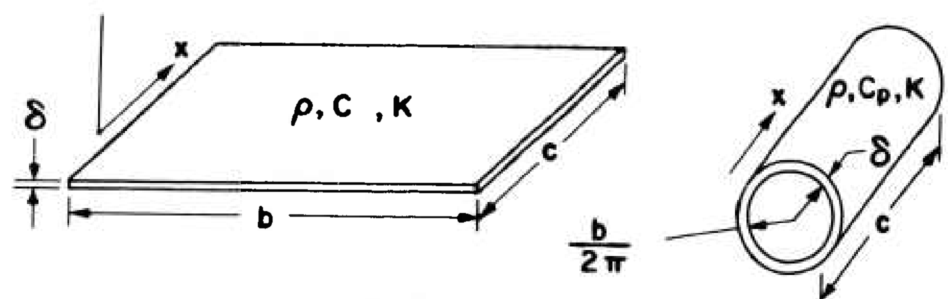
THE THERMAL RESPONSE OF AN INSULATED PLATE OR CYLINDER TO A HEAT IMPULSE APPLIED UNIFORMLY ALONG ONE EDGE

A.1 Introduction

The notion that changes of temperature within a body are dependent on time is clear cut. Exactly what this dependence is is often obscured. In order to show this dependence in a relatively simple manner, a one-dimensional example has been chosen which indicates how long it takes for a thermal shock, as reflected by the temperature distribution, to reach equilibrium.

A.2 Mathematical Formulation of the Problem

Consider the plate of span b, chord c, and thickness δ; or the circular cylindrical shell of radius $b/2\pi$, length c, and thickness δ, shown in Sketch A.1.



SKETCH A.1

All surfaces and edges of either configuration are assumed to be adiabatic walls except for one edge of each (the edge of each figure shown in the sketch which is nearest the reader) which is adiabatic except when a heat impulse, Q, is applied to it.

The material of the structure is assumed to have properties as follows which are constant: density ρ, specific heat c_p, and thermal conductivity K. The thermal diffusivity, a^2, is defined as

$$\alpha^2 = \frac{K}{\rho C_p} \tag{A. 1}$$

Since the thermal analysis of both configurations is identical, we proceed by considering the heat conduction equation

$$\dot{T} = \alpha^2 \nabla^2 T \tag{A. 2}$$

The temperature is only a function of the space variable x. It does not vary through the thickness or, in the case of the plate, along the span. Equation (A. 2) may be written

$$\dot{T} = \alpha^2 T'' \tag{A. 3}$$

where

$$\dot{T} = \partial T/\partial t \tag{A. 4}$$

and

$$T'' = \frac{\partial^2 T}{\partial x^2} \tag{A. 5}$$

The equation is variable separable and the solution is assumed in the form

$$T(x, t) = \sum_{i=0}^{\infty} T_i(t)\, X_i(x) \tag{A.6}$$

Substituting each term of the series composing the right hand side of Eq. (A.6) into Eq. (A.3) yields the following

$$\frac{\dot{T}_i}{T_i} = \alpha^2 \frac{X_i''}{X_i} = -\lambda_i \tag{A.7}$$

or

$$\dot{T}_i + \lambda_i T_i = X_i'' + \frac{\lambda_i}{\alpha^2} X_i = 0 \tag{A.8}$$

Equations (A.8) are satisfied by the following expressions for $T = \sum_{i=0}^{\infty} T_i X_i$

$$T_i = e^{-\lambda_i t}\left[A_i \cos\sqrt{\frac{\lambda_i}{\alpha^2}}\, x + B \sin\sqrt{\frac{\lambda_i}{\alpha^2}}\, x\right] \tag{A.9}$$

The constants λ_i, A_i, and B_i are specified acccrding to the boundary conditions and initial conditions. The assumption that the edges of the plate (or cylinder) are insulated may be stated

$$T'(0, t) = T'(\varsigma, t) = 0 \tag{A.10}$$

Substituting Eq. (A. 9) into Eqs. (A. 10) one finds

$$\lambda_i = \frac{i^2 \pi^2 \alpha^2}{c^2} \tag{A. 11}$$

and

$$B_i = 0 \tag{A. 12}$$

Consequently, Eqs. (A. 9) reduce to the set

$$T_i = A_i \, e^{-\left(\frac{i \pi \alpha}{c}\right)^2 t} \cos \frac{i \pi x}{c} \tag{A. 13}$$

Recalling that

$$T(x, t) = \sum_{i=0}^{\infty} T_i \tag{A. 14}$$

and substituting Eq. (A. 13) into Eq. (A. 14) one obtains a convenient expression for the temperature

$$T = A_o + \sum_{n=1}^{\infty} A_n \, e^{-\left(\frac{n \pi \alpha}{c}\right)^2 t} \tag{A. 15}$$

The coefficients A_o, A_n must be determined by considering the initial conditions. At $t = 0$ the temperature distribution is given by [according to Eq. (A. 15)]

$$T(x, o) = A_o + \sum_{n=1}^{\infty} A_n \cos \frac{n \pi x}{c} \tag{A. 16}$$

The introduction of a quantity of heat Q at the "leading" edge, in the form of an impulse, does not alter the fact that the temperature is zero every where but at the leading edge. The temperature distribution everywhere but at the leading edge is thus

$$T(x,0) = 0 \qquad\qquad 0 < x \leq c \tag{A. 17}$$

The temperature singularity existing at the leading edge is best described by the following Stiltjes integral which is arrived at by consideration of the principle of conservation of (thermal) energy. Consequently the initial temperature distribution is completely specified by the equations

$$\lim_{\Delta x \to 0} \int_0^{\Delta x} T(x,0)\,dx = \frac{Q}{\rho C_p b \delta}$$

$$T(x,0) = 0 \qquad\qquad 0 < x \leq c \tag{A. 18}$$

The initial temperature distribution as given above may be expressed as a linear combination of the space dependent eigenvectors of the partial differential equation and the aforementioned boundary conditions. In other words, the coefficients A_o, A_n appearing in Eq. (A. 16) are determined by the temperature distribution as given above in Eqs. (A. 18). The linear combination of eigenvectors is readily recognized as a simple Fourier cosine series. The coefficients A_o, A_n of this series are readily determined by the usual relations for coefficients of a Fourier series. This yields

$$A_o = \lim_{\Delta x \to 0} \frac{1}{c} \int_0^{\Delta x} T(x,0)\,dx \tag{A. 19}$$

and

$$A_n = \lim_{\Delta x \to 0} \frac{2}{c} \int_0^{\Delta x} T(x,0) \cos \frac{n\pi x}{c} \, dx$$ (A. 20)

These two equations yield

$$A_o = \frac{Q}{\rho c_p c b \delta}$$

$$A_n = \frac{2Q}{\rho c_p c b \delta}$$

$$n = 1, 2, 3 \cdots$$ (A. 21)

Substituting Eqs. (A. 21) into Eq. (A. 15), we have the final desired result.

$$T(x,t) = \frac{Q}{\rho c_p c b \delta} \left[1 + 2 \sum_{n=1}^{\infty} e^{-\left(\frac{n\pi\alpha}{c}\right)^2 t} \cos \frac{\pi n}{c} x \right]$$ (A. 22)

A. 3 Conclusions and Some Numerical Examples

The results obtained from Eqs. (A. 22) are shown in graphical form in Figure A. 1. In this figure the quantity $T(x,t)$ divided by $Q/\rho c_p c b \delta$ has been plotted versus the nondimensional characteristic length x/c. The nondimensional temperature distribution is shown at six separate times.

$$t_o = 0 \;, \quad t_1 = \left(\frac{c}{\alpha\pi}\right)^2 \;, \quad t_2 = 2\left(\frac{c}{\alpha\pi}\right)^2$$

$$t_3 = 3\left(\frac{c}{\alpha\pi}\right)^2, \quad t_4 = 4\left(\frac{c}{\alpha\pi}\right)^2, \quad t_5 = \infty$$ (A. 23)

Referring to the figure, one sees that by time t_3 the temperature distribution caused by this severe shock has pretty well smoothed out. t_1 is chosen arbitrarily as the characteristic time of this element. The characteristic time is given as

$$\bar{t} = \left(\frac{c}{\alpha \pi}\right)^2 \tag{A. 24}$$

The important conclusion to be drawn from this analysis is that as long as the heat pulse is distributed uniformly over the surface of application, the temperature distribution depends on only one distance parameter which may be termed the characteristic length. This characteristic length is the length of the configuration along the direction of the heat flow. In the examples cited this dimension is c. (See Sketch A. 1) The analysis could just as well have been done by considering the heat pulse to be applied uniformly to the surface. In this latter instance the characteristic length would be δ.

It is interesting to compare the "characteristic times" for a typical plate. One characteristic time is associated with application of a heat pulse along the leading edge and the other characteristic time is associated with application of a heat pulse to the upper or lower surface of the plate (inner or outer surface in the case of a cylinder). For a stainless steel plate with the following dimensions the characteristic times t_{1c} and $t_{1\delta}$ are given below. These are the times it would take a temperature distribution caused by a heat impulse applied along the leading edge and along one surface, respectively, to attain the shape identified as t_1 in Figure A. 1.

$b = 22$ ft.

$c = 7$ ft.

$\delta = 8$ in.

$K = 2.18 \times 10^{-4}$ BTU/in. sec. $^{\circ}F$

$\rho = 0.286$ lbs/in^3

$c_p = 0.118$ BTU/lbs$^{\circ}F$

$t_{1c} = 31.06$ hours

$t_{1\delta} = 16.73$ minutes

As a result of the above, it seems plausible to assume that no significant change will occur in a temperature distribution during a correcting, noncritical return to the specified flight path, and certainly the time for a significant change to occur is longer than any characteristic vibration period.

APPENDIX B

STATISTICAL GUST LOADING CRITERION

Reference 18 suggests a design criterion for a heated airplane from the probability of failure view point. When the gust velocity criterion is applied to an airplane which is at elevated temperatures during part of its flight, the problem that arises is the determination of the temperature to use in evaluation of material allowables for design. If the highest temperature attainable is used in the calculation of allowables, then this will certainly result in a safe design. However, a more logical and less stringent requirement would be to make the chance of failure of an airplane encountering gusts at elevated temperature equal to an established probability criterion. This probability criterion could be established by calculating the probability of failure of an airplane, designed by existing gust criterion which encounters gusts at room temperature only. It is then possible, by a trial and error procedure to find the temperature that will make the probability of failure the same. This temperature is the design temperature.

The following is a procedure of estimating the probability of failure as a result of encountering gust loads. This method is subject to certain simplifying assumptions. These are

1. The gust loads can be represented as a stationary time series of a normal distribution, where the root mean square value varies in intensity.

2. The alleviation factor and the transfer function can be calculated using only one condition of Mach number and altitude.

3. The design load factor is assumed to be the result of gusts at this altitude and Mach number.

4. The stress is proportional to the load factor. The temperature distribution is for a point of maximum stress.

For a stationary time series of a normal distribution, the number of maximum values of $y(t)$ per second exceeding any given value y_o is given by

$$N(y_o) = \frac{1}{2\pi}\left[\frac{\int_0^\infty \omega^2 \phi(\omega)\, d\omega}{\int_0^\infty \phi(\omega)\, d\omega}\right] e^{-y_o^2/2\sigma^2} \qquad \text{(B. 1)}$$

where

$y(t)$ = a random variable

ω = frequency

$\phi(\omega)$ = power spectrum of $y(t)$

σ = root mean square value of $y(t)$

By letting $y(t)$ be the acceleration of the airplane due to gusts, it is possible to find the number of peak accelerations per second above a given value "a". When the root mean square value of gust velocity is varying as in the case of actual turbulence, the number of peak accelerations per second in excess of "a" is given by

$$N(a) = N_o \int_0^\infty \hat{f}(\sigma_u)\, e^{-a^2/2(\bar{A}\sigma_u)^2} \qquad \text{(B. 2)}$$

where

$$N_0 = \frac{1}{2\pi}\left[\frac{\int_0^\infty \omega^2 \phi_u(\omega) A^2(\omega)\, d\omega}{\int_0^\infty \phi_u(\omega) A^2(\omega)\, d\omega}\right]$$

is the total number of peak acceleration per second of all magnitudes.

$\hat{f}(\sigma_u)$	is the distribution function of the root mean square gust velocity.
σ_u	is the root mean square gust velocity.
$A(\omega)$	is the transfer function of the airplane.
$\bar{A}$	$= \frac{1}{\sigma_u}\left[\int_0^\infty \phi_u(\omega) A^2(\omega)\, d\omega\right]^{1/2}$
$\phi_u(\omega)$	is the power spectrum of the gust velocity.

With these equations it is possible to determine the number of accelerations above any given value. If the given value of acceleration is the limit value that the airplane is designed to withstand, then the number of accelerations above this value will be the probability of failure. If the design limit acceleration is high, the probability of failure is low. And conversely, if the design limit acceleration is low, the probability of failure is high.

The method of establishing the equivalent probability criterion corresponding to an established discrete gust criterion is as follows.

First, find the maximum acceleration the airplane must withstand to survive the discrete gust of the specified magnitude. The discrete gust will usually be a fifty foot per second gust of a one-minus-cosine profile. The value of the maximum acceleration will be found using one altitude and

Mach number. Then, calculate the number of accelerations per second the airplane will experience in excess of this maximum acceleration. This number is the probability of failure criterion.

To illustrate the criterion, consider the following example. Find the design criteria for a rigid airplane subjected to gust loads and elevated temperatures. Select the design temperature and load factor so as to make the chance of failure of the above airplane the same as the chance of failure of an airplane (different in strength but similar in all other respects to the above airplane) designed to encounter the same gust loads at room temperature.

Let the airplane designed for gust and elevated temperatures be airplane number one. This airplane is designed by load factor a_{d1} applied at room temperature or by load factor a_{d2} applied at the design temperature T_d. Note that when either a_{d1} or the T_d is found the problem is solved.

Let the airplane designed for gust loads only be airplane number two. This airplane is designed by load factor a_{d2} applied to the airplane at room temperature. This problem assumes that a_{d2} has been calculated. This may be found by methods of the established gust criteria applied at the altitude and Mach number of interest.

If airplane number two is designed by load factor a_{d2}, then the probability that a_{d2} is exceeded, or in short, failure of the airplane is given by

$$N(a_{d_2}) = N_0 \int_0^\infty \hat{f}(\sigma_v) e^{-a^2_{d_2}/2(\bar{A}\sigma_v)^2} d\sigma_v \qquad \text{(B. 3)}$$

The elevated temperature effects the structure by reducing the allowable stress. Define

$$S_a = \eta(T) S_A \tag{B.4}$$

S_A = the allowable stress in the structure at room temperature

S_a = the allowable stress at temperature

$\eta(T)$ = the factor of degradation of allowable stress.

When designing a structure, it is necessary to specify the design load and the design allowables. Thus, the same structure would result if one designed airplane number one at room temperature for the design load factor a_{d1} or if one designed airplane number one at T_1 (by "at T_1" is meant designed using the allowable $S_{a1} = \eta(T_1) S_A$ instead of the allowable S_A) subject to the new load factor $\eta(T_1) a_{d1}$. This new load factor would be less since $\eta(T_1)$ is less than one for $T_1 > T_{\text{room temperature}}$.

During the life of the airplane the fraction of the total time that the temperature of the point of interest is between T_n and T_{n-1} is ρ_n and, if the total number of temperature divisions is R, then

$$\sum_{n=1}^{R} \rho_n = 1 \tag{B.5}$$

Consider the chance of failure during the time when $T \cong T_n$. Then the design load factor which corresponds to this temperature is $\eta(T_n) a_{d1}$ and since the gust loads are independent of the airplane temperature the chance of failure is

$$N\{n(T_n)a_{d_1}\} = \rho_n N_0 \int_0^{\infty} \hat{f}(\sigma_v) e^{\frac{-[n(T_n)a_{d_1}]^2}{2(\bar{A}\sigma_v)^2}} d\sigma_v \tag{B.6}$$

or the total probability of exceeding the design conditions is

$$N'(a_{d_1}) = \sum_{n=1}^{R} \rho_n N_0 \int_0^{\infty} \hat{f}(\sigma_v) e^{\frac{-[n(T_n)a_{d_1}]^2}{2(\bar{A}\sigma_v)^2}} d\sigma_v \tag{B.7}$$

Then from the statement of the problem we make $N(a_{d2}) = N'(a_{d1})$ or

$$\sum_{n=1}^{R} \rho_n N_0 \int_0^{\infty} \hat{f}(\sigma_v) e^{\frac{-[n(T_n)a_{d_1}]^2}{2(\bar{A}\sigma_v)^2}} d\sigma_v = N_0 \int_0^{\infty} \hat{f}(\sigma_v) e^{\frac{-(a_{d_2})^2}{2(\bar{A}\sigma_v)^2}} d\sigma_v \tag{B.8}$$

Thus in Eq. (B.8) we know all but a_{d1}. Thus solve Eq. (B.8) for a_{d1}. This is done by a trial and error method. The integrals involved must be evaluated numerically.

It has been shown that it is possible to arrive at the same structure two different ways; first, by using room temperature allowables and a design load factor a_{d1}, or second, by using the design load

factor of a_{d2} and material properties at some elevated temperature, the design temperature (T_d).

Then T_d is the solution of

$$\eta(T_d) = \frac{a_{d_2}}{a_{d_1}} \tag{B.9}$$

This leads to the new design criteria for aircraft number one.

1. Design load factor a_{d2}.
2. Design temperature equals T_d (temperature at which the allowables are calculated.)

T_d represents a weighted temperature penalty that results in a design aircraft number one with the same chance of failure as aircraft number two.

Thus, the method outlined above gives a rational method of weighing the added hazard of encountering a gust at elevated temperature. This method will lead to a lower design requirement than the straight superposition of gust load plus the maximum obtainable structure temperature. At the same time the structure will be as safe as an airplane not subjected to high temperature which is designed by present gust criterion.

APPENDIX C

EQUILIBRIUM TEMPERATURE CONSIDERATIONS

C.1 Introduction

The dependence of thermal stresses on temperature distributions occurring in structures requires a precise description of the temperatures themselves before any determination of the stresses can be made. While aerodynamic heating is the primary cause of high temperature aircraft structural problems, considerable alleviation of these stresses and/or resultant deflections may be present at higher Mach numbers due to the smoothing out of temperature distributions due to thermal radiation.

The detailed analysis of how temperature distributions might be alleviated by thermal radiation should consist of two parts: the first part demonstrating the reduction in temperature at a particular point attributable to thermal radiation and the second part showing how the temperature gradients in the structure are smoothed out. The second part is not attempted in this analysis as it is a function of a specific structural configuration.

To demonstrate how much local temperatures can be lowered as a result of radiation a simple model has been chosen and is described below. Figure C. 1 shows the results of the calculations based on this model.

C.2 The Mathematical Model

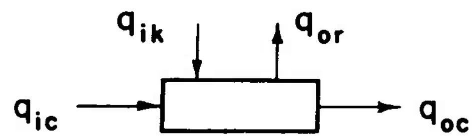
The model consists of a thin (temperatures are constant through the thickness) flat plate "flying" at zero angle of attack in the earth's atmosphere. For the purposes of these calculations the plate is permitted to fly at altitudes of from 0 to 90,000 feet and at a variety of Mach numbers ranging between Mach 1 and Mach 6.

The temperature at a point one foot aft of the location where the flow over the plate changes from laminar to turbulent is determined from the requirement that the convective heat transfer to the particular point on the plate is equal to the radiative heat loss from that point per unit time. The temperature of the point determined in this way is termed the equilibrium temperature, T_w.

Consider the differential element of a plate shown in the sketch below. If the element is in thermal equilibrium, the principle of conservation of energy requires that

$$q_{in\ conductive} + q_{in\ convective} = q_{out\ radiative} + q_{out\ conductive}$$

$$q_{ik} + q_{ic} = q_{or} + q_{oc} \tag{C. 1}$$



SKETCH C-I CONSERVATION OF HEAT ENERGY IN A DIFFERENTIAL ELEMENT OF A PLATE

It is assumed that the heat transferred by conduction is small when compared to the other modes of heat transfer. This permits the following simplification. The radiation heat loss is given by the Stefan-Boltzmann law, namely,

$$q_{or} = \varepsilon \sigma T_w^4 \tag{C. 2}$$

In order to show the maximum temperature alleviation the emissivity, ϵ, is considered unity. The convective heat transfer to the system is described in terms of the heat transfer coefficient, h, the adiabatic wall temperature, T_{aw}, and the equilibrium temperature of the point in question, T_w.

$$q_{in} = h(T_{aw} - T_w) \tag{C.3}$$

In reference 22 the heat transfer coefficient for the turbulent region of the boundary layer is given as

$$h = x^{-1/5}\, 0.0296\, k\left(\frac{u}{\nu}\right)^{4/5} P_r^{1/3} \left(T/T_w\right)^{0.45} \tag{C.4}$$

Assuming air behaves as a perfect gas the adiabatic wall temperature may be written

$$T_{aw} = T\left[1 + r\left(\frac{\gamma-1}{2}\right)M^2\right] \tag{C.5}$$

The heat balance equation thus becomes

$$\left[x^{-1/5}\, 0.0296\, k\left(\frac{u}{\nu}\right)^{4/5} P_r^{1/3} \left(T/T_w\right)^{0.45}\right]$$

$$\left[T\left(1 + r\,\frac{\gamma-1}{2}M^2\right) - T_w\right] = \sigma T_w^{4} \tag{C.6}$$

Equation (C.6) must be solved for T_w. It should be noticed that the temperature at any point x on the flat plate in the turbulent region of the boundary layer may be obtained from this equation. In this example, T_w is obtained for a value of $x = 1$ foot and for various Mach numbers ranging from 1 to 6 and altitudes from 0 to 90,000 feet. Once the distance x has been chosen, we have a two parameter system — altitude and Mach number being the parameters to be varied. All the quantities appearing in Eq. (C.6) which depend on altitude or Mach number are first written in terms of altitude and Mach number and then these expressions are introduced into Eq. (C.6). With this accomplished the equation may be written simply

$$c_1 + c_2 T_w = T_w^4 \tag{C.7}$$

Altitude and Mach number are varied for the computation of c_1 and c_2; then Eq. (C.7) is solved by trial and error for T_w using a high speed digital computer.

The essential features of the problem are presented above. The equations which permit the reduction of Eq. (C.6) to Eq. (C.7) are given below.

C.3 Determination of Various Parameters in Terms of Mach Number and Static Temperature

The parameters appearing in the expressions which determine the equilibrium temperature can be written most conveniently in terms of Mach number and the static air temperature. This is done as follows. Since the equilibrium temperature is required for various Mach numbers and altitudes, the altitude is chosen first and then the static air temperature is determined from the appropriate relation between air temperature and altitude, depending on whether the location is in the troposphere or stratosphere.

The thermol conductivity of air, K, expressed in units of BTU per hour-foot-degrees Rankine may be written as (Ref. 23)

$$K = 0.001091 \; T^{3/2} (T + 362\,°R)^{-1} \tag{C. 8}$$

The velocity of the airstream u may be written (Ref. 24) in terms of the Mach number, the speed of sound and the absolute temperature.

$$u = C_o \left(T/T_o \right)^{1/2} M \tag{C. 9}$$

The air density in the troposphere may be written (Ref. 24)

$$\rho_T = \rho_o \left(T/T_o \right)^{4.2561} \tag{C. 10}$$

The air density in the strotosphere is given as follows.

$$\rho_S = \rho_o \log_{10}^{-1} \left[-0.527139 - B\left(\frac{T_x - T}{a} \right) \right] \tag{C. 11}$$

The kinemotic viscosity, ν, is defined as

$$\nu = \mu / \rho \tag{C. 12}$$

Consequently, from the above result, the kinematic viscosity in the troposphere, ν_T, may be written

$$\nu_T^{-1} = 4.4053\,\rho_o\,(T/T_o)^{4.2561}\,T^{-3/2}\,(T + 198.7\,°R)\,10^{7} \tag{C. 13}$$

and in the stratosphere the expression becomes

$$\nu_S^{-1} = \rho_o \log_{10}^{-1}\left[-0.527139 - B\left(\frac{T_* - T}{a}\right)\right]$$
$$\left[4.4053\,\rho_o\,(T/T_o)^{4.2561}\,10^{7}\,T^{-3/2}\,(T + 198.7\,°R)\right]$$

(C. 14)

where $$B = \frac{G \log_{10} e}{R\,T_*}$$

Combining the previous expressions, the equations for the equilibrium temperature may be written.

<u>For the troposphere — 35,000 feet and below</u>

$$X^{-1/5}\,0.0296\left\{0.001091\;T^{3/2}\,(T + 362\,°R)^{-1}\right\}$$
$$\left\{C_o\,(T/T_o)^{1/2}\,\dot{M}\;4.4053\,\rho_o\,(T/T_o)^{4.2561}\,10^{7}\right.$$
$$\left.T^{-3/2}\,(T + 198.7\,°R)\right\}^{4/5}\,P_r^{1/3}\left(\frac{T}{T_w}\right)^{0.45}$$
$$\left\{T\left(1 + r\,\frac{\gamma - 1}{2}\,M^2\right) - T_w\right\} = \sigma\,T_w^{4} \tag{C. 15}$$

For the stratosphere — 40,000 feet and above

$$X^{-1/5}\, 0.0296 \left\{ 0.001091\, T^{-3/2} (T + 362\,^\circ R)^{-1} \right\} \left\{ c_0 \left(T/T_0 \right)^{1/2} \right.$$

$$M \rho_0 \log_{10}^{-1} \left[-0.527139 - B \left(\frac{T_* - T}{a} \right) \right] \Big[4.4053\, \rho_0$$

$$\left. (T/T_0)^{4.2561}\, 10^{7}\, T^{-3/2} (T + 198.7\,^\circ R) \Big] \right\}^{4/5}$$

$$Pr^{1/3} \left(\frac{T}{T_w} \right)^{0.45} \left\{ T \left(1 + \frac{\gamma - 1}{2} M^2 \right) - T_w \right\} = \sigma T_w^{4}$$

(C. 16)

Equations (C. 15) and (C. 16) are of the form

$$C_1 + C_2 T_w = T_w^{4}$$

(C. 17)

C_1 and C_2 are functions of the altitude, the Mach number and the location of the point in question on the plate as detailed above. Figure C. 1 shows the equilibrium temperature at a point one foot behind the transition point for various Mach numbers and altitudes. Superimposed on this plot are the adiabatic wall temperatures corresponding to these altitudes and Mach numbers. The constants used in calculating the values for Figure C. 1 are listed in Table C. 1.

It will be observed that at the point considered the radiation heat loss is of little significance below Mach 3. On the other hand, at Mach 5 and 90,000 feet, the temperature is reduced from 2105 °R to 1390 °R, while at sea level and Mach 5, the temperature is reduced from 2790 °R to 2465 °R.

The importance of including the thermal radiation term in temperature analyses of aircraft structures (above Mach 3, for external aerodynamic heating) is demonstrated clearly by the above analysis. Because of the linear dependence of elastic thermal stresses on the temperature one would expect an alleviation of thermal stresses corresponding to the alleviation in the temperature distribution. Since the thermal stresses occurring in a structure depend on the entire spatial temperature distribution, and since the above analysis considered the temperature at only one point, the whole effect of alleviating the thermal stresses is not presented. However, similar alleviation should be found if an entire structure is considered.

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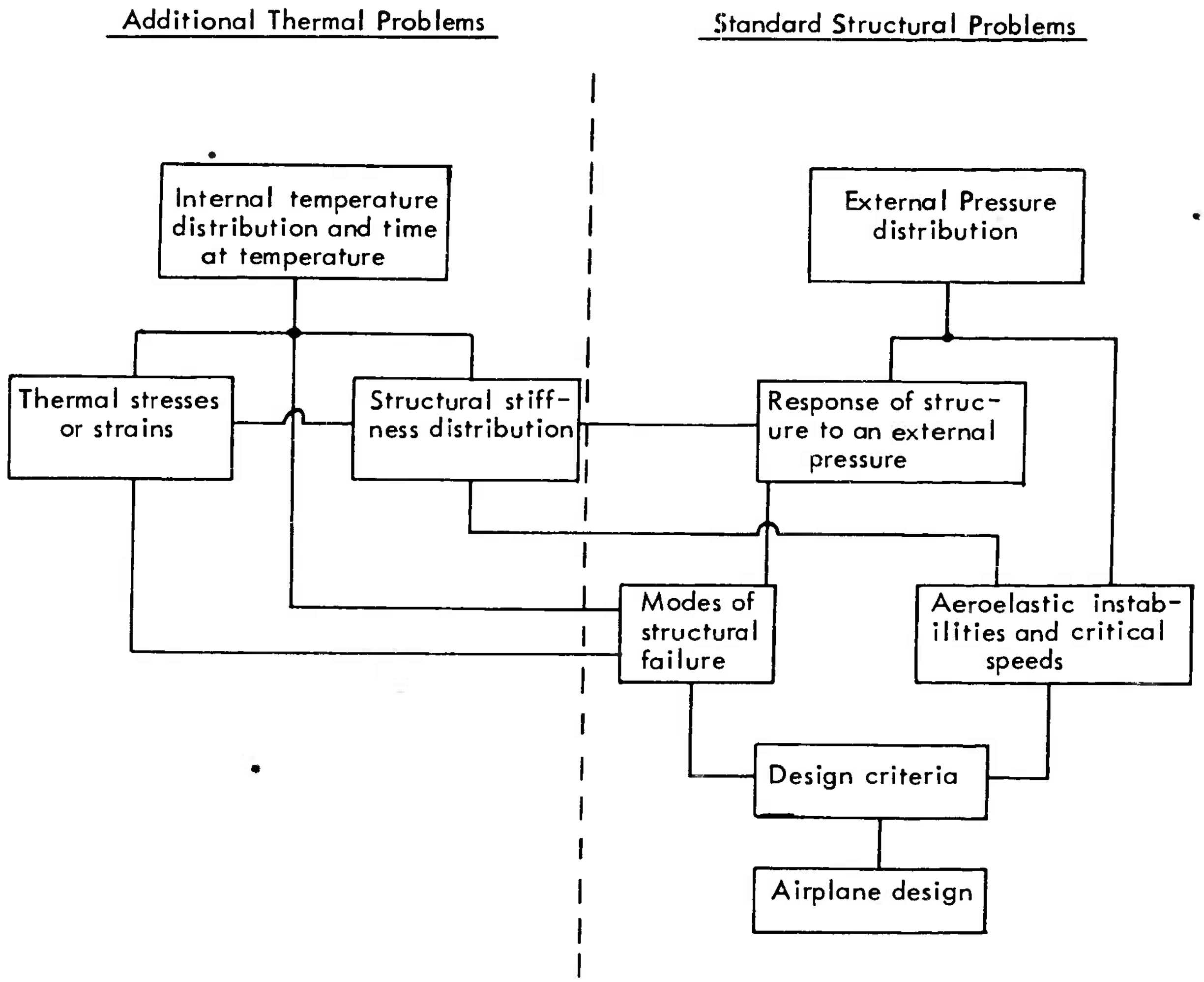

FIG. 2.1 — STRUCTURAL ANALYSIS BLOCK DIAGRAM

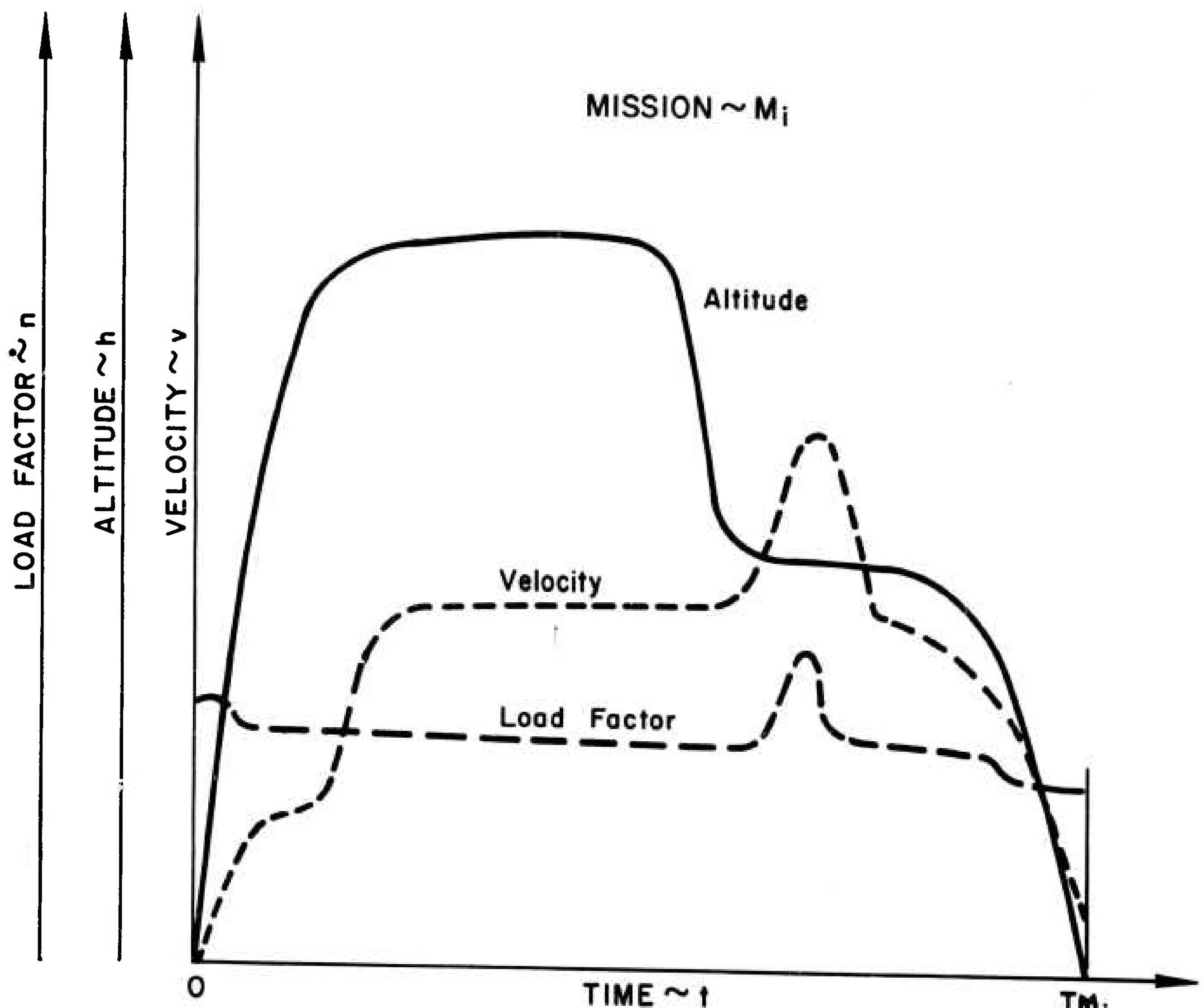

FIG. 4.1 TYPICAL MISSION TIME HISTORY

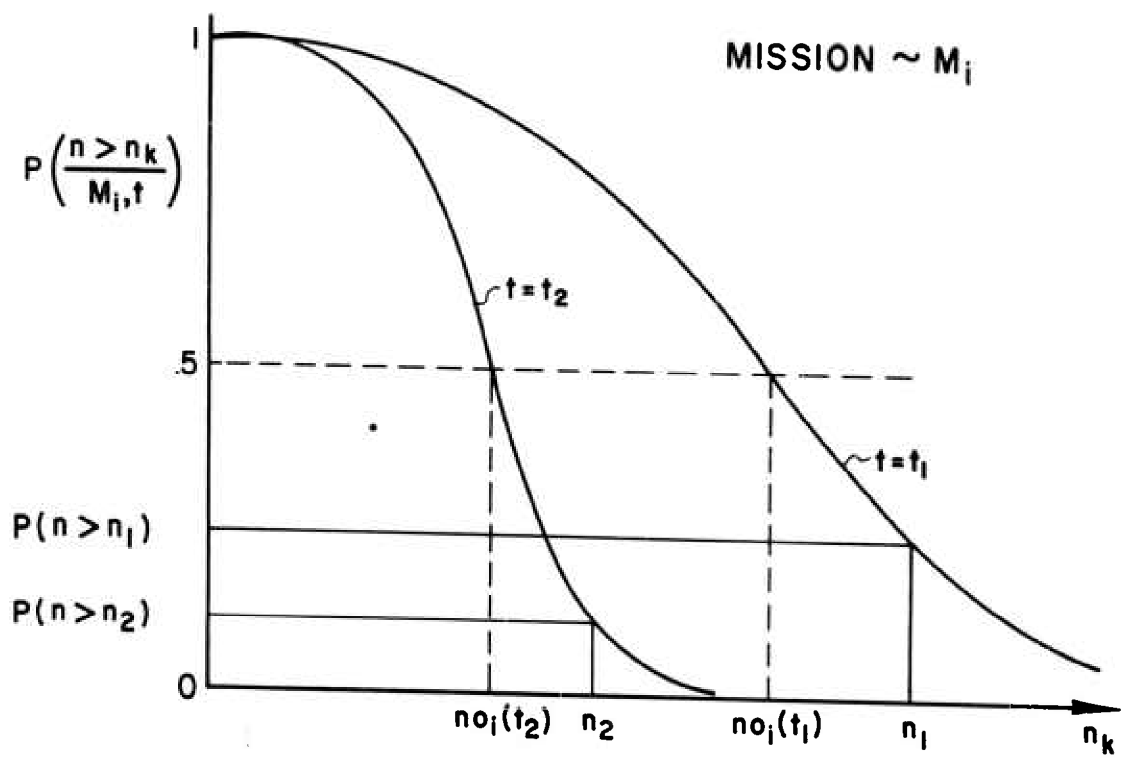

FIG. 4.2 PROBABILITY OF EXCEEDING A LOAD FACTOR n_k AT TWO PARTICULAR TIMES DURING A GIVEN MISSION

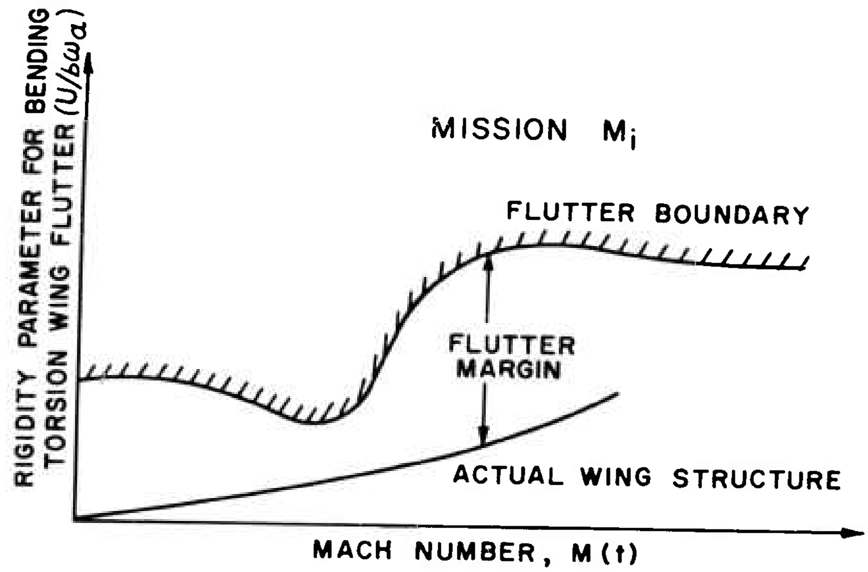

FIG. 4.3 FLUTTER MARGIN FOR AIRCRAFT SUBJECTED TO AERODYNAMIC HEATING

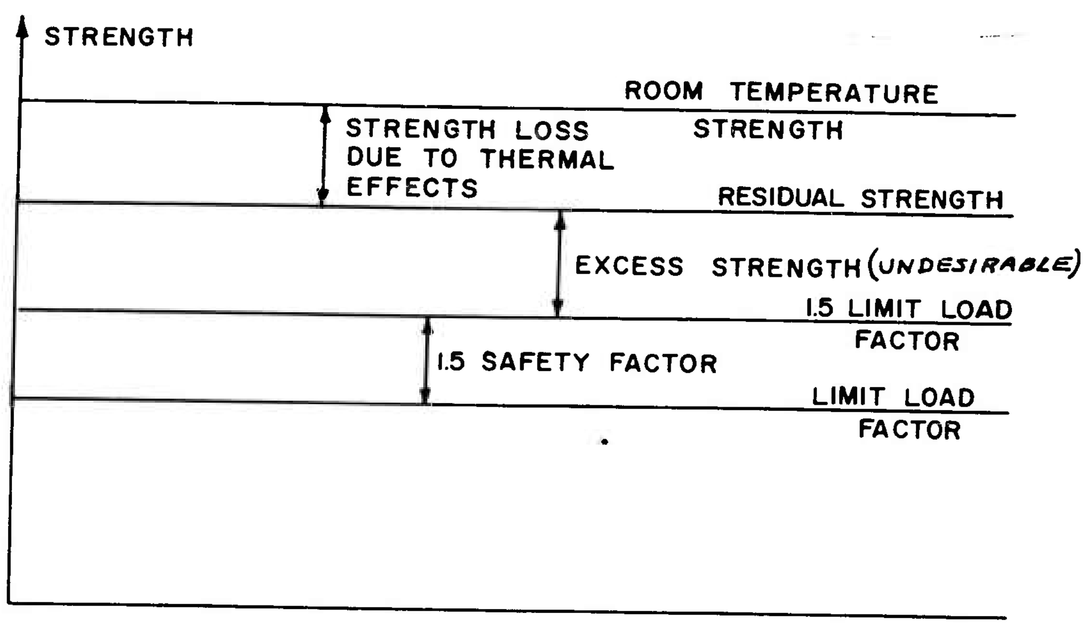

FIG. 5.1 ILLUSTRATION OF STRENGTH DESIGN REQUIREMENT

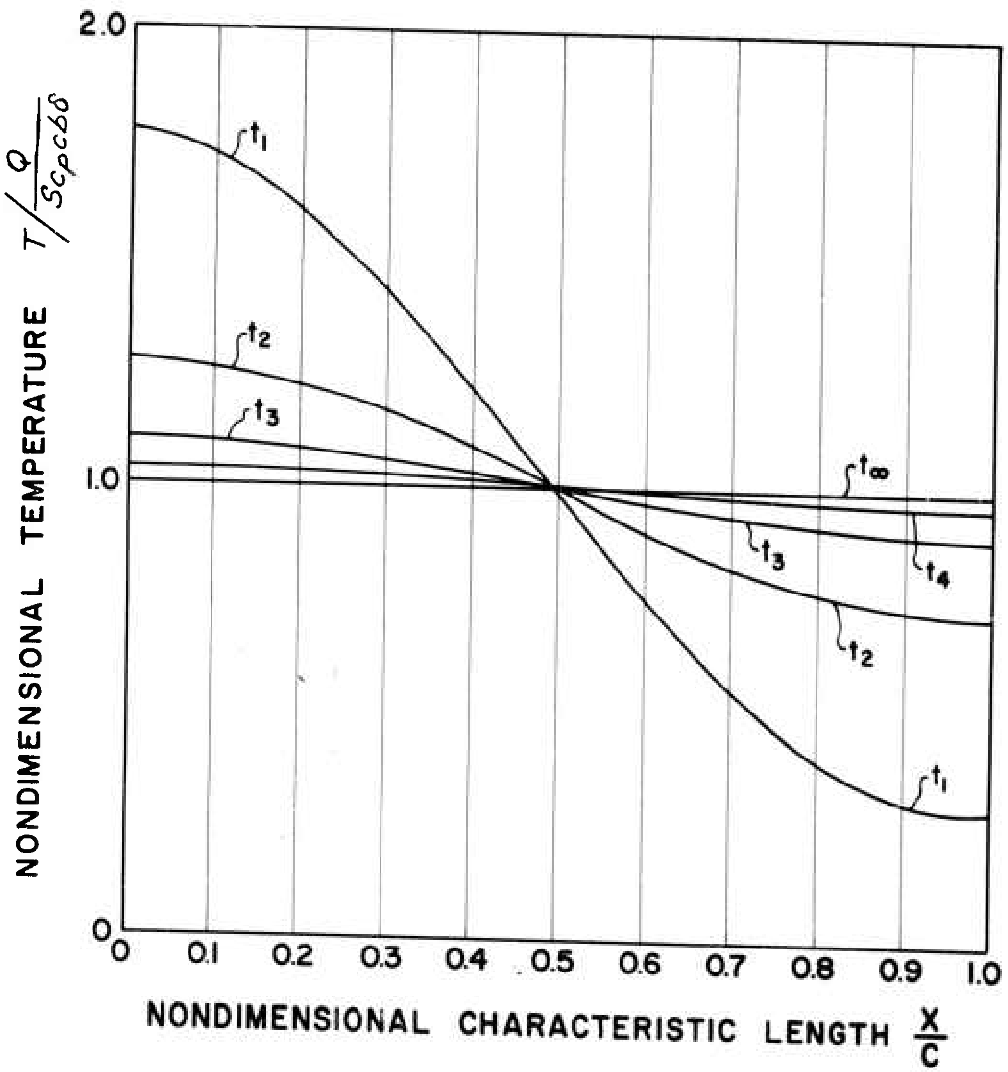

FIG. A-1 TEMPERATURE DISTRIBUTION IN ONE-DIMENTIONAL STRUCTURE

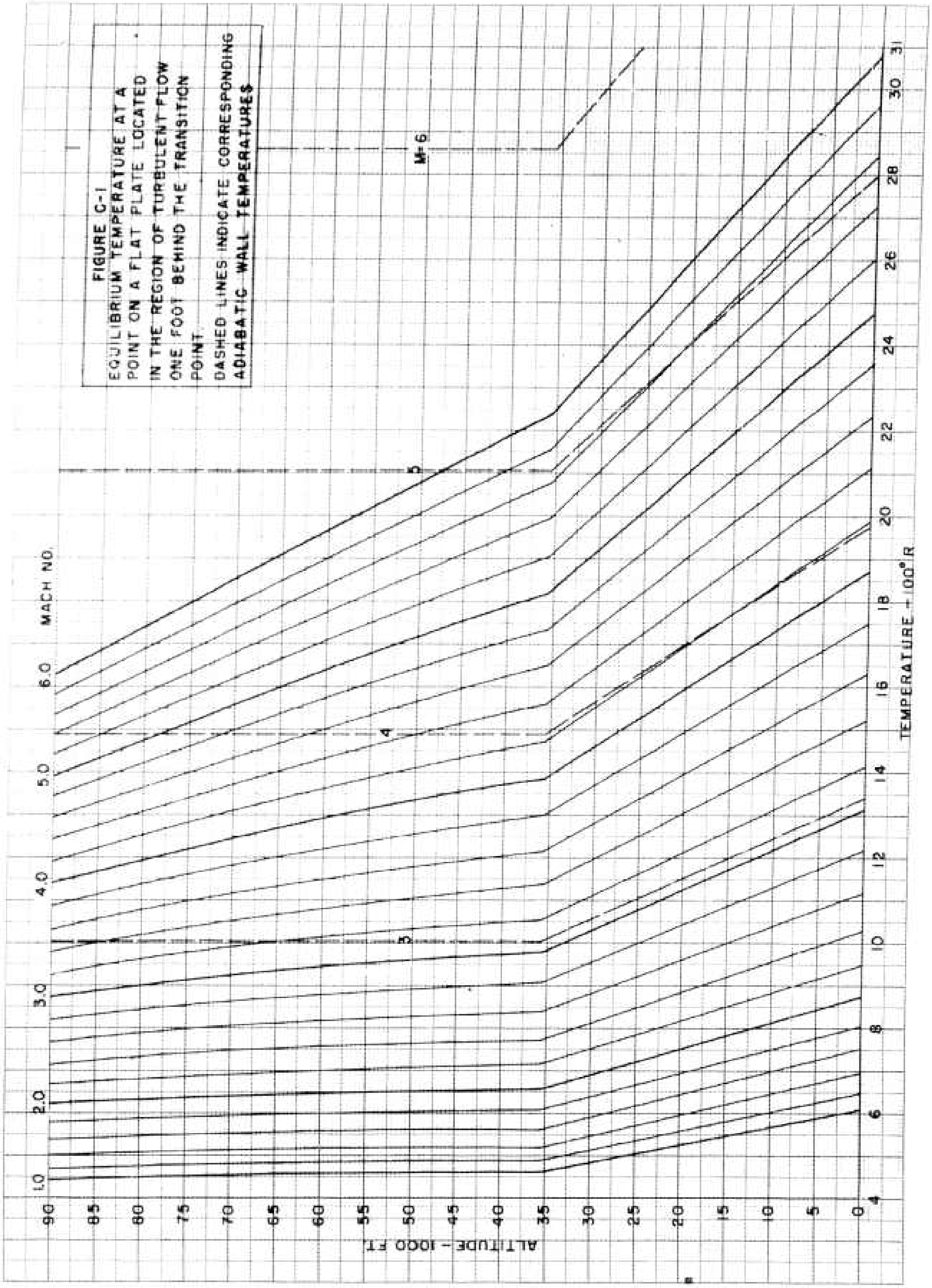

FIGURE C-1
EQUILIBRIUM TEMPERATURE AT A POINT ON A FLAT PLATE LOCATED IN THE REGION OF TURBULENT FLOW ONE FOOT BEHIND THE TRANSITION POINT.
DASHED LINES INDICATE CORRESPONDING ADIABATIC WALL TEMPERATURES

TABLE C. 1

CONSTANTS USED IN DETERMINING FIGURE C. 1

Parameter	Symbol	Value
Velacity of saund at sea level	C_o	1116.89 ft/sec
Temperature at sea level	T_o	518.688 ^oR
Recovery factor	r	0.88
Emissivity	ε	1
Stefan-Boltzmann canstant	σ	0.17310 $BTU/ft^2hr\ ^oR$
Temperature lapse rate	a	$3.56616 \times 10^{-3}\ ^oR/ft$
Dimensianal canstant	G	32.17405 ft^2
Trapopause temperature	T^*	389.988 ^oR
Gas canstant	R	1716.49 $ft^2/sec^2\ ^oR$